Eidgenössische Technische Hochschule Zürich
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Shrimps predict bioinformatics failures

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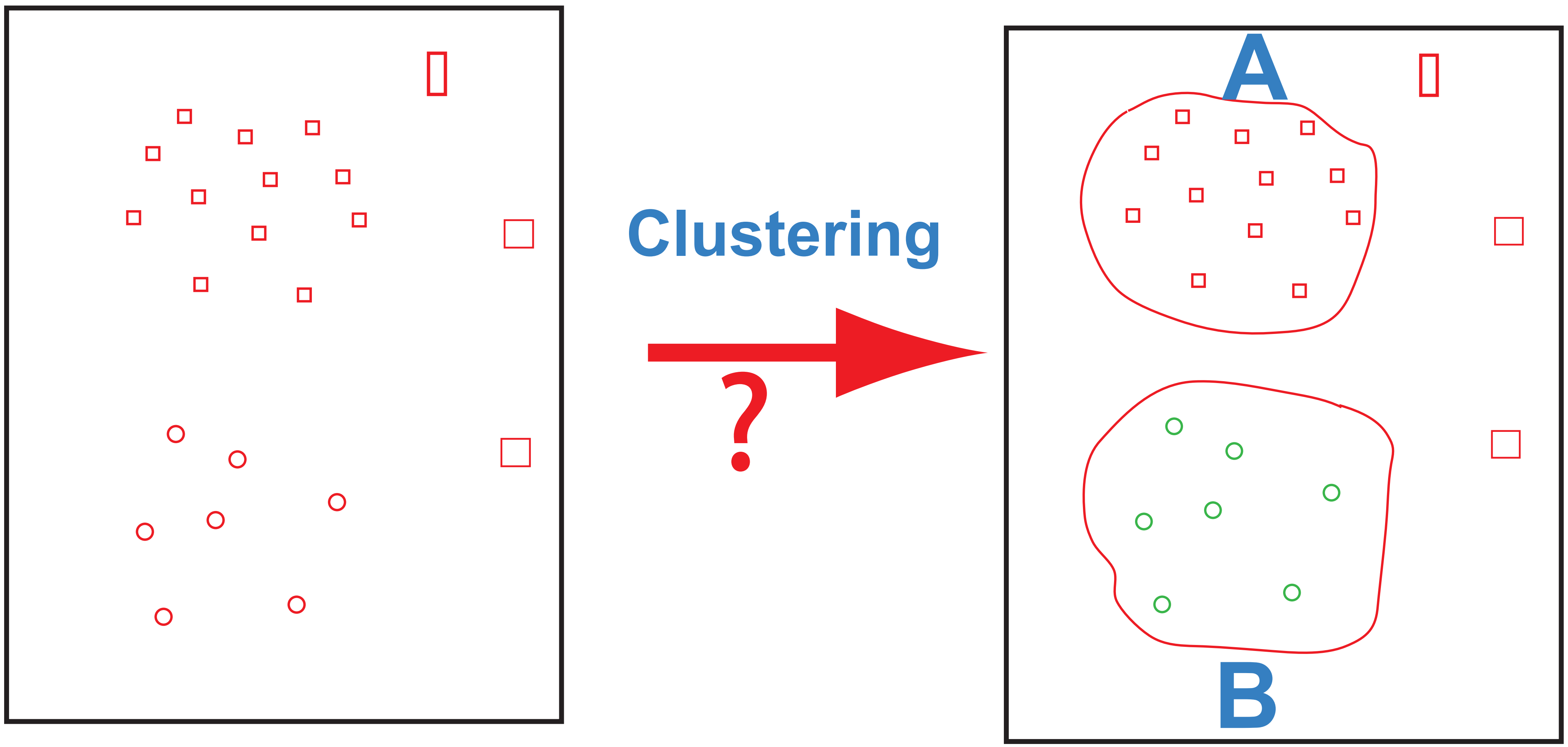
Overview

- Introduction
- The shrimps phenomenon
- Experimental evidence of shrimps in electronic circuits
- Shilnikov ordering of shrimps and beyond
- Internal mechanisms of shrimp generation
- Global shrimp organization: spiral vs. linear
- Clustering relevance

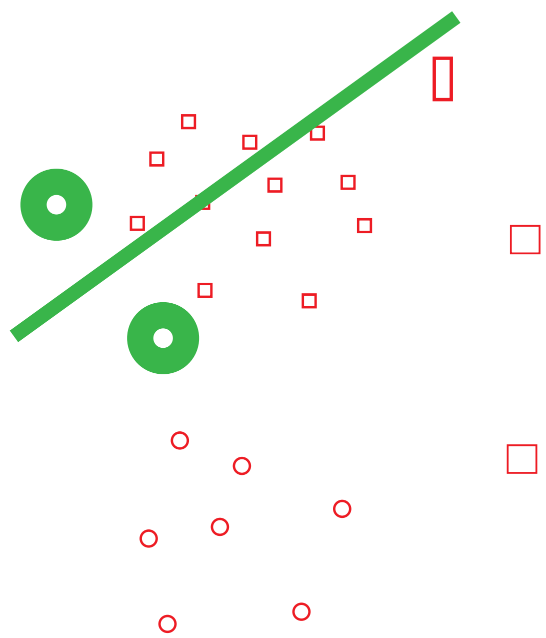
I. Problem setting:



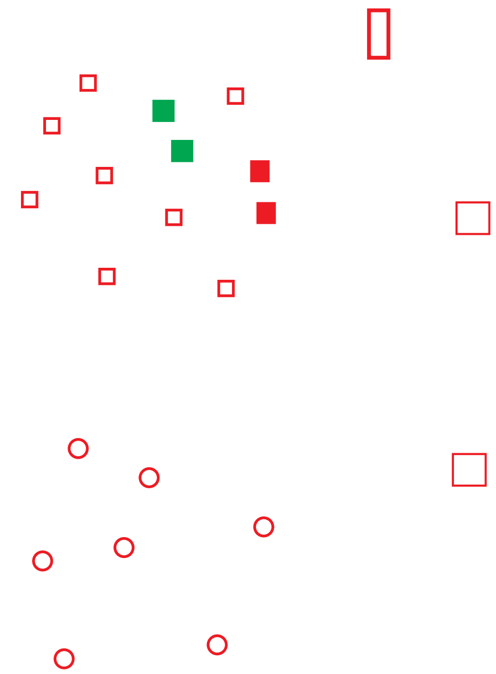
: basic perceptual / cognitive process, important to bioinformatics

Prominent 'solutions':

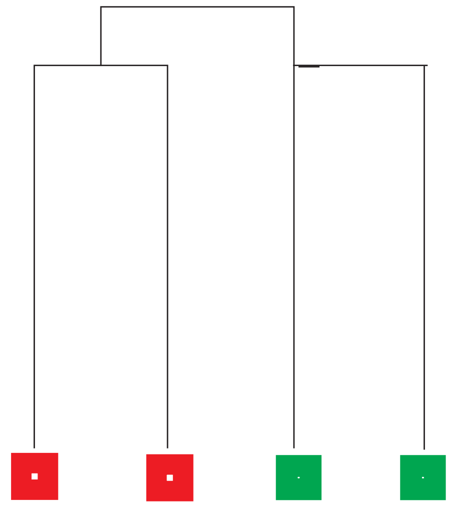
: K-means



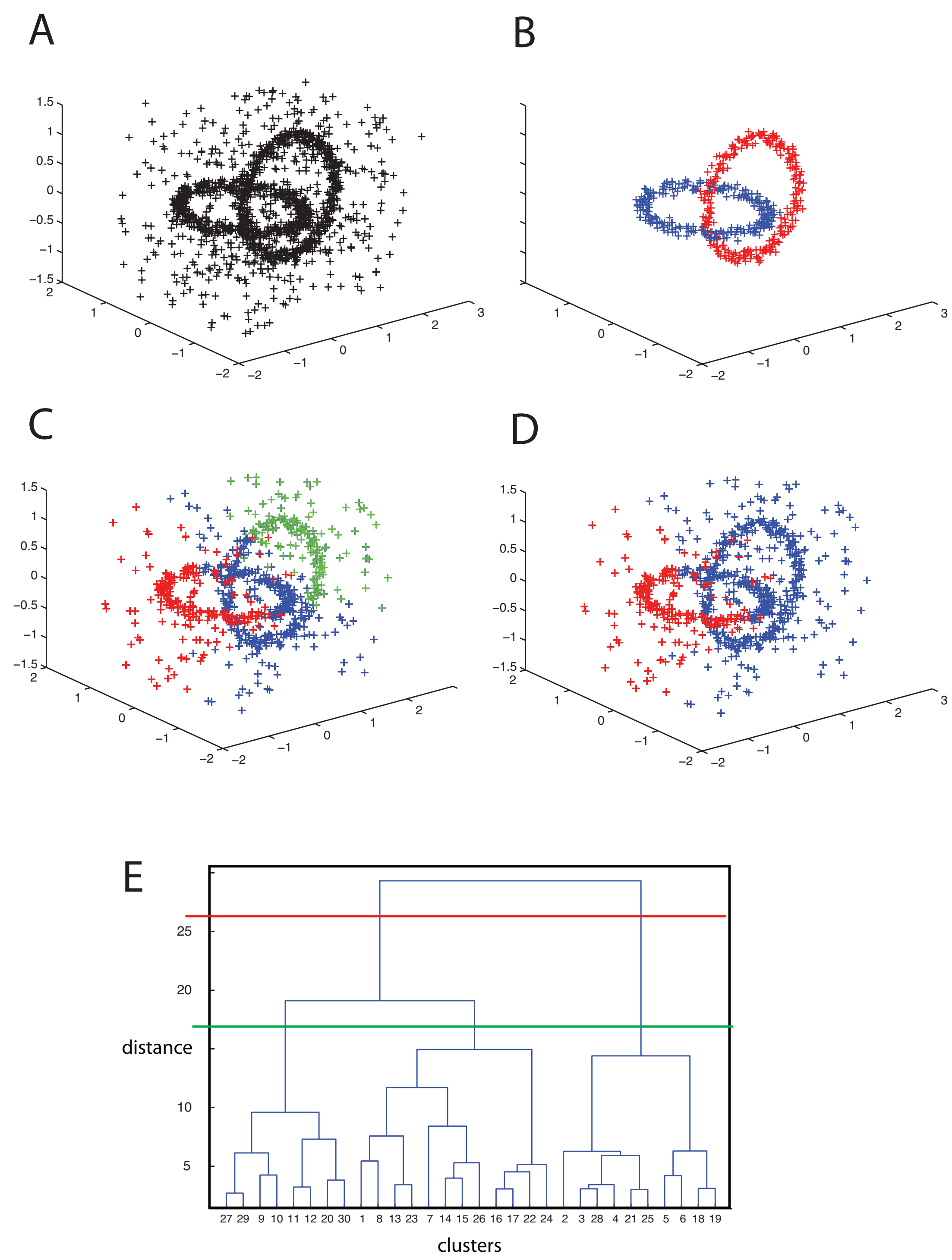
: Ward's clustering



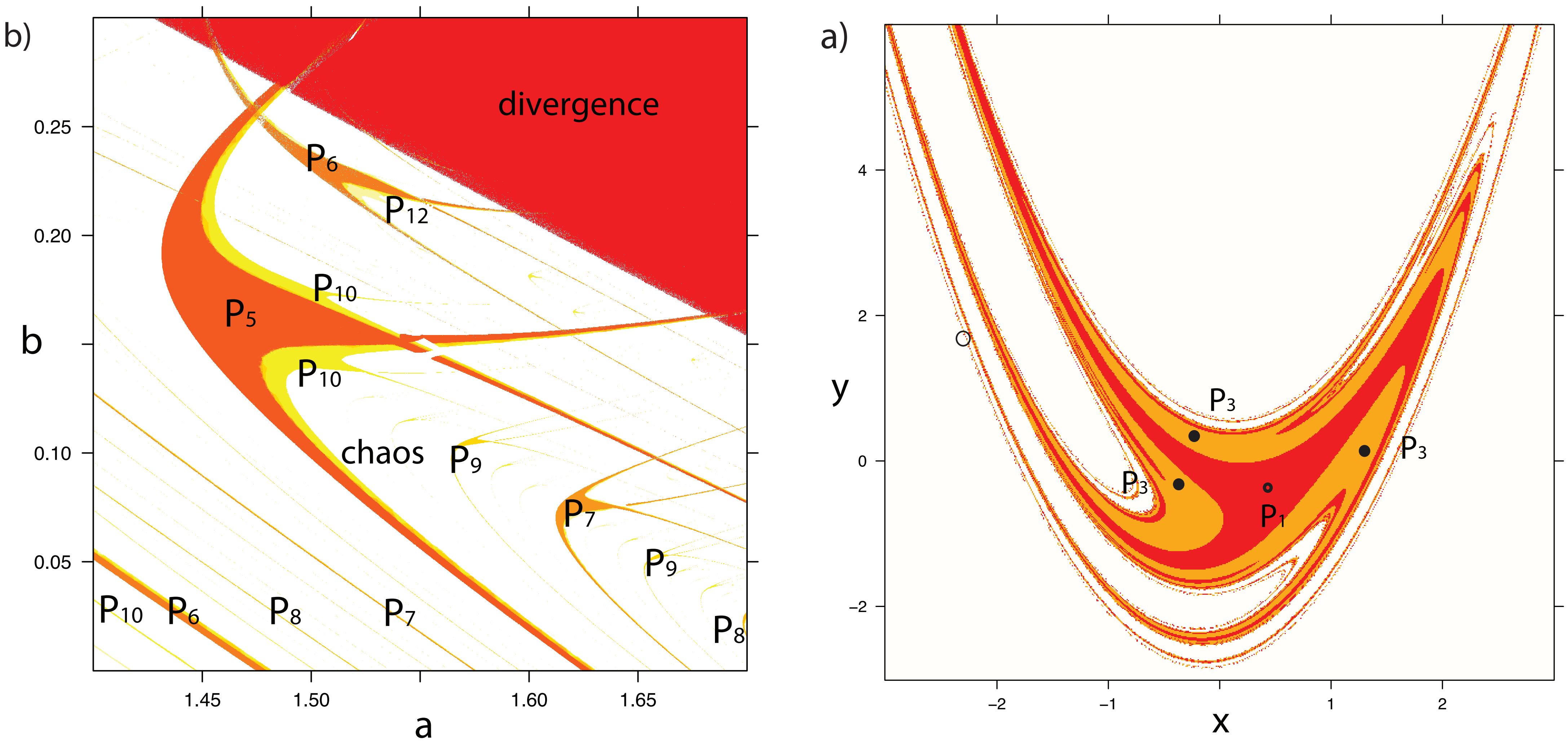
dendrogram



Problems:

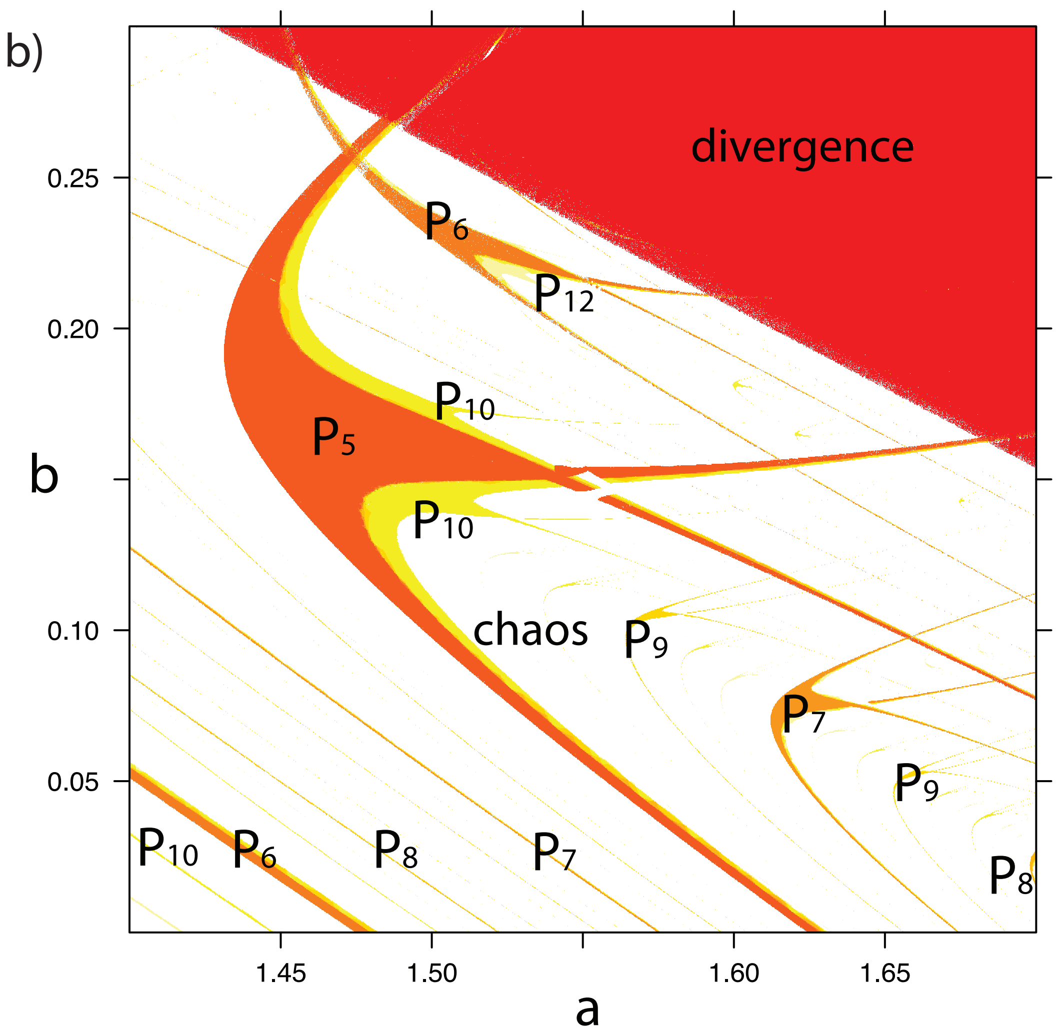


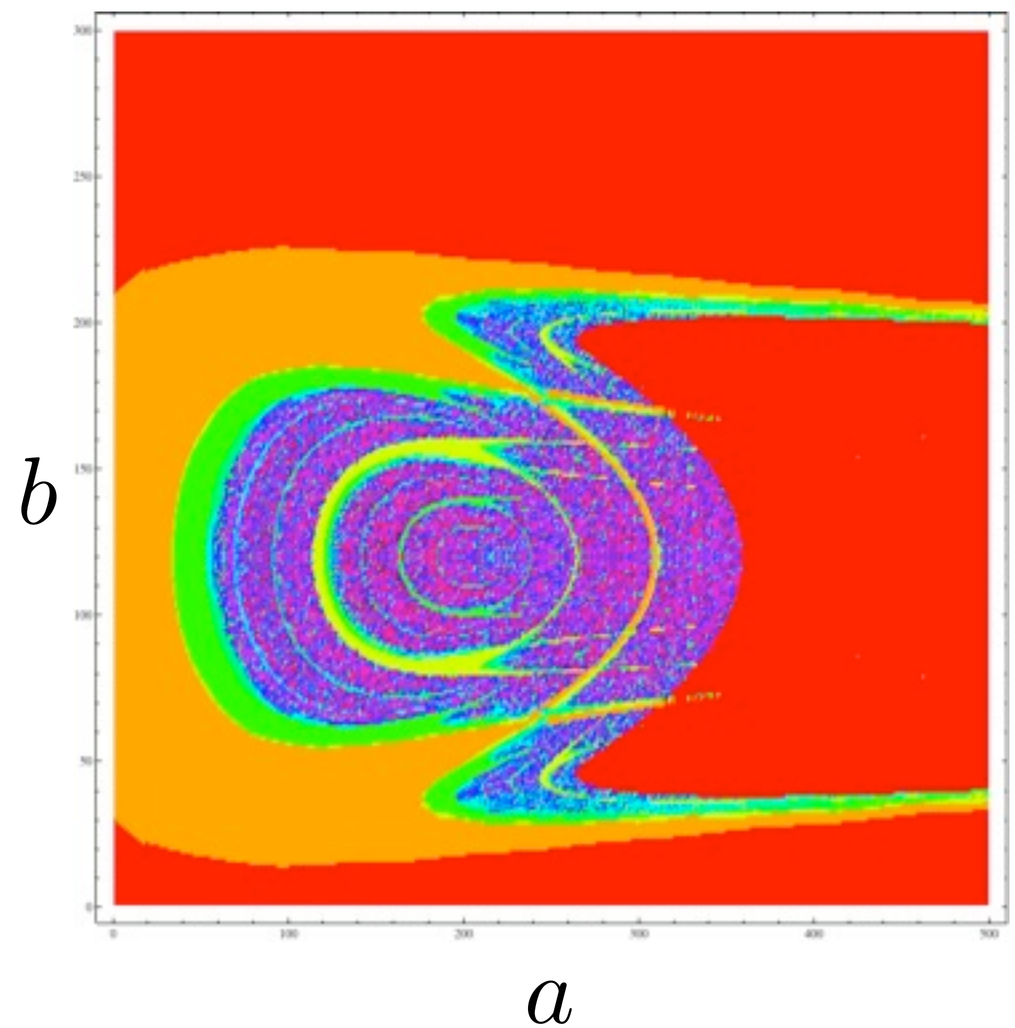
II. ‘Shrimps’ in the Henon system:



$$f(\{x,y\}) := \{-ax^2 + y + 1, bx\}; \quad a = 1.4; b = 0.3$$ **‘strange attractor’**

...in general maps:

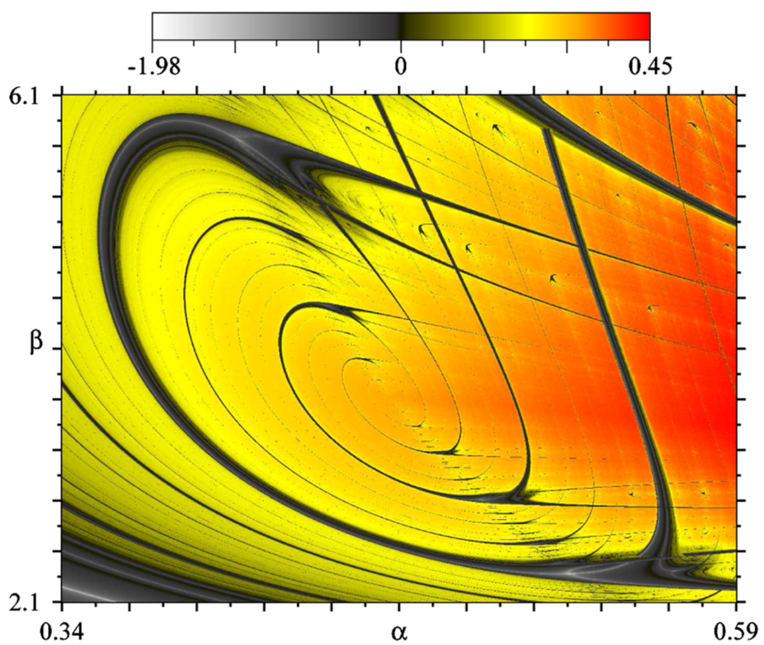


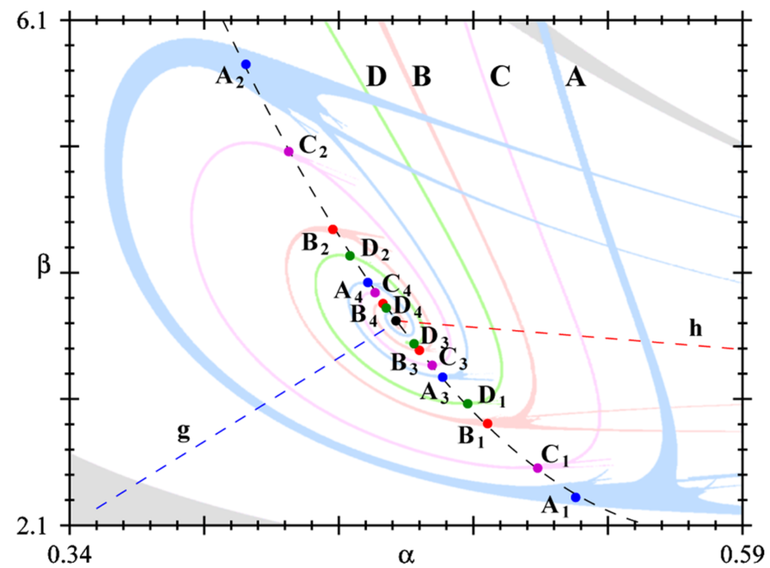


Dissipative Henon map

$$f(x) = \frac{(2 \times 3^{1/2} + a)(1 - (2 \times 3^{1/2} + a)^2 x^2}{3!} + \frac{(5/6 + b^2)(2 \times 3^{1/2} + a)^4 x^4}{5!}$$

Shrimps in ODE's: organized along spirals:





PRL **101,** 054101 (2008) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2008

Periodicity Hub and Nested Spirals in the Phase Diagram of a Simple Resistive Circuit

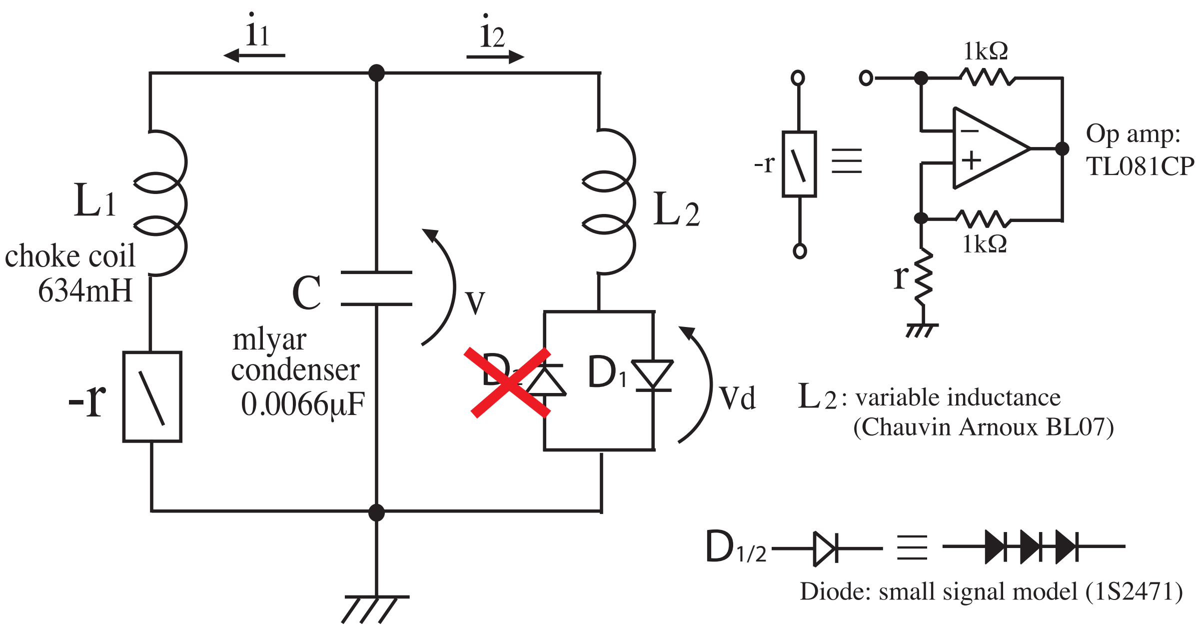
Cristian Bonatto and Jason A. C. Gallas

We report the discovery of a remarkable "periodicity hub" inside the chaotic phase of an electronic circuit containing two diodes as a nonlinear resistance. The hub is a focal point from where an infinite hierarchy of nested spirals emanates. By suitably tuning two reactances simultaneously, both current and voltage may have their periodicity increased continuously without bound and without ever crossing the surrounding chaotic phase. Familiar period-adding current and voltage cascades are shown to be just restricted one-parameter slices of an exceptionally intricate and very regular onionlike parameter surface centered at the focal hub which organizes all the dynamics.

Our aim here is to describe the striking organization around a remarkable parameter point, an organizational hub, discovered inside the chaotic phase of a circuit studied

Finally, we mention that it is very tempting to associate spiral nestings with the much studied homoclinic orbits. However, numerical work shows spirals not to exist in some flows which are textbook examples of the Shilnikov setup. We described the unfolding of an infinite sequence of spirals in the vicinity of the numerically found focal hub. We believe our investigation to be accurate albeit not rigorous, and remark that we are not aware of any theory to predict and locate hubs.

III. Are these Real-World phenomena?



$$\dot{x} = \alpha x + z, \quad \dot{y} = z - f(y), \quad \dot{z} = -x - \beta y.$$

$$f(y) = \frac{\gamma}{2}\left(\left| y + \frac{1}{\gamma} \right| - \left| y - \frac{1}{\gamma} \right| \right), \quad \text{(diode)}$$

'Nishio-Inaba circuit', symmetric

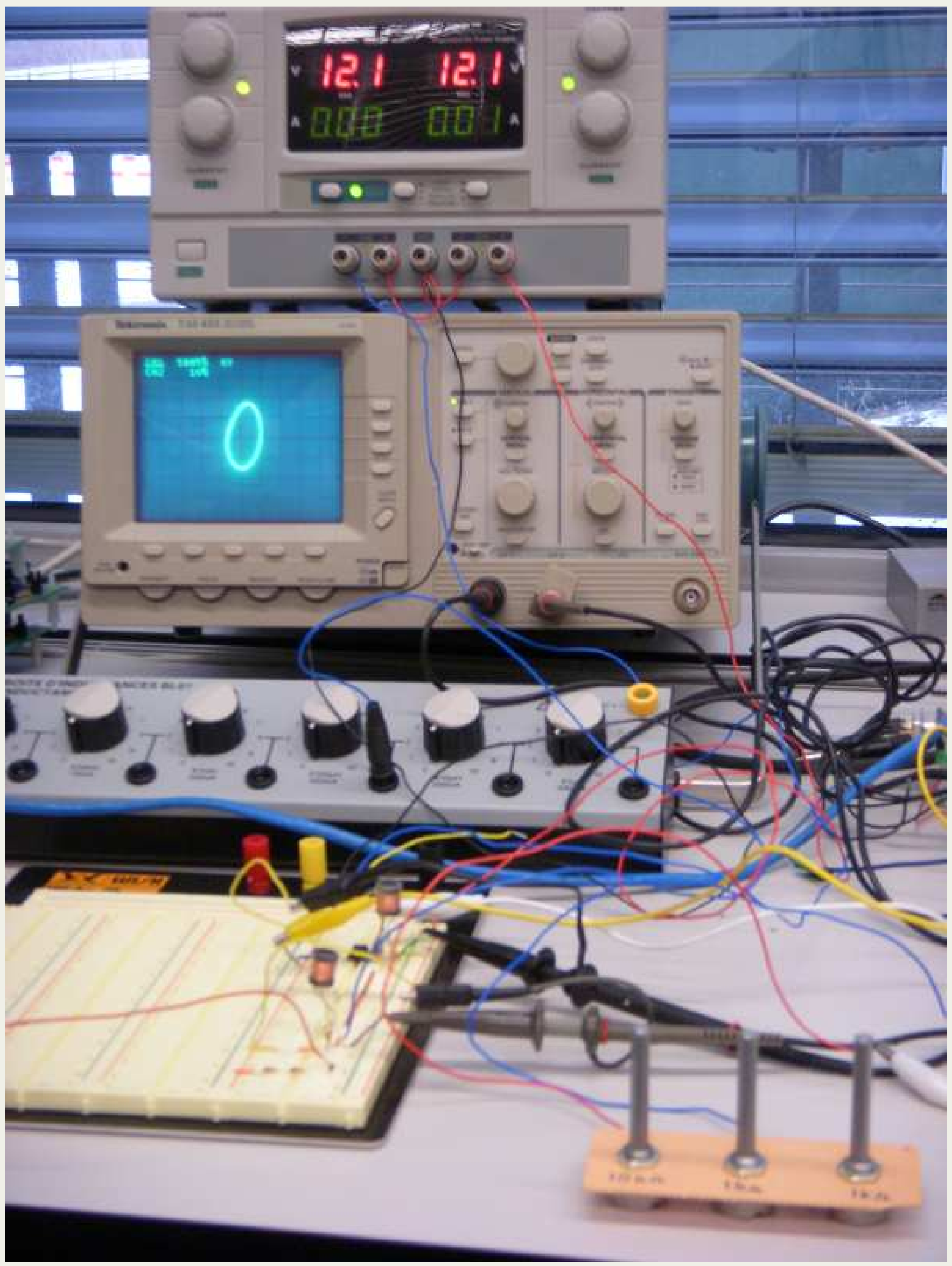
✗ asymmetric circuit

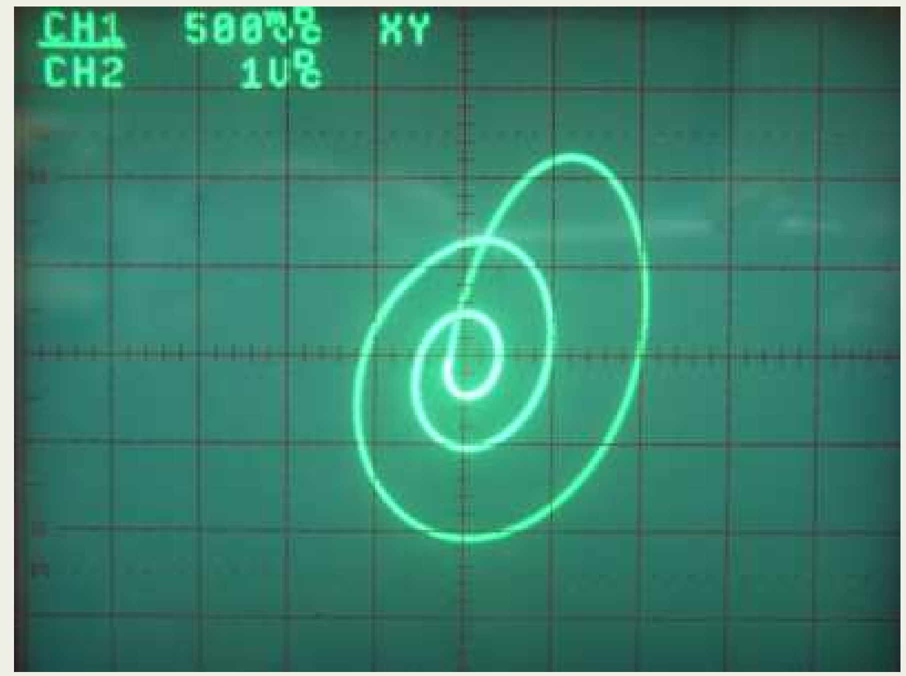
Theoretical works:

- Bifurcation Phenomena near Homoclinic Systems:
A Two-Parameter Analysis
P. Gaspard, R. Kaprai, and G. Nicolis , J. Stat. Phys.1984

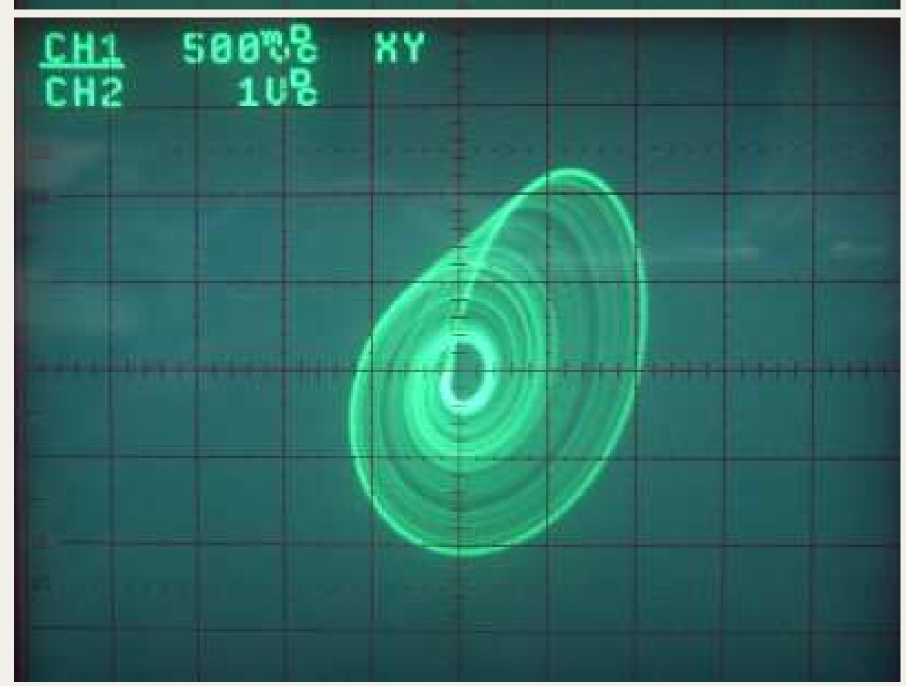
- Rigorous Anlyses of Windows in a Symmetric Circuit
Y. Nishio, N. Ianba, S. Mori, and T. Saito, IEEE TRANS. CIRC.SYS. 37, 1990

Experiment:



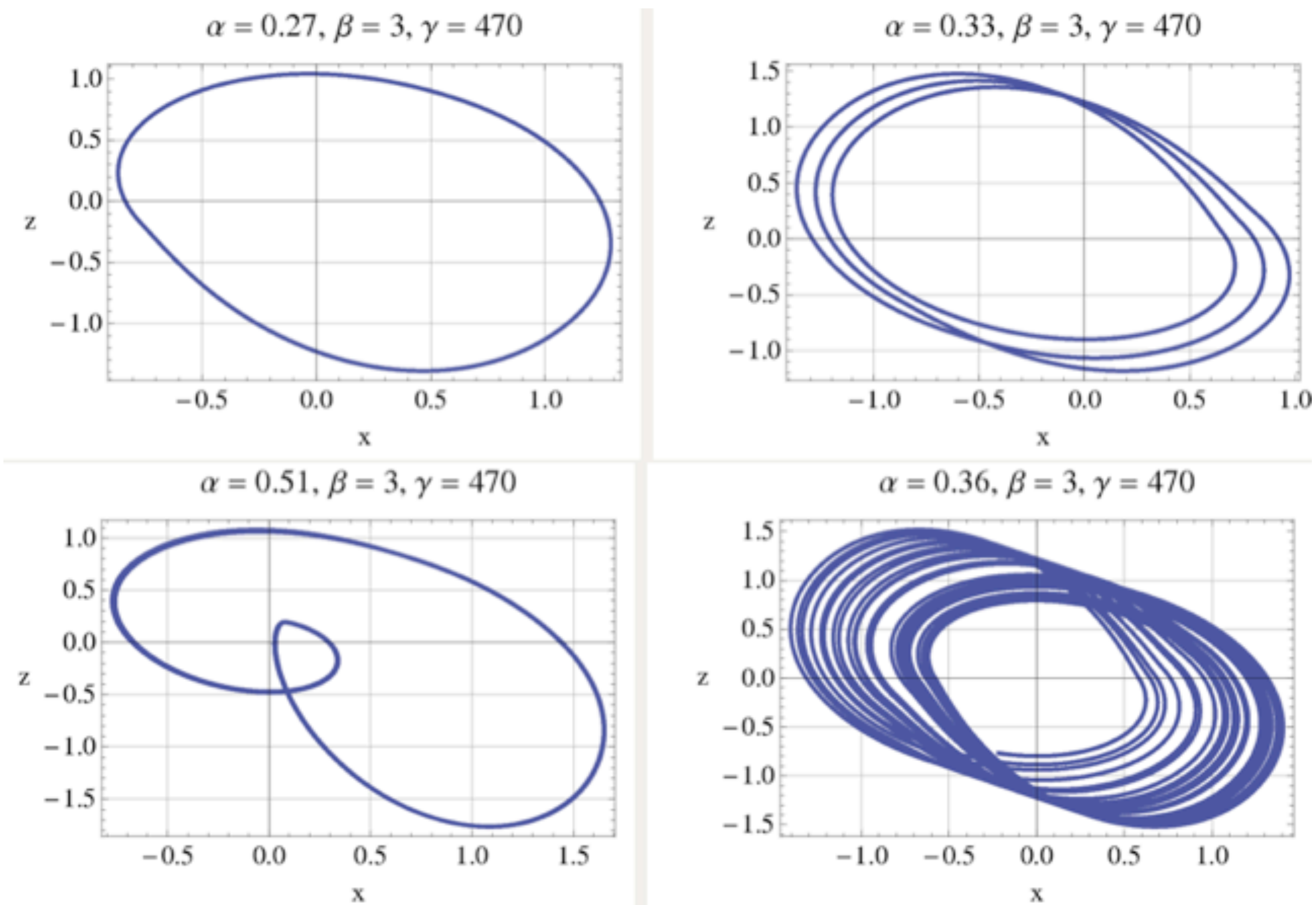


period-3

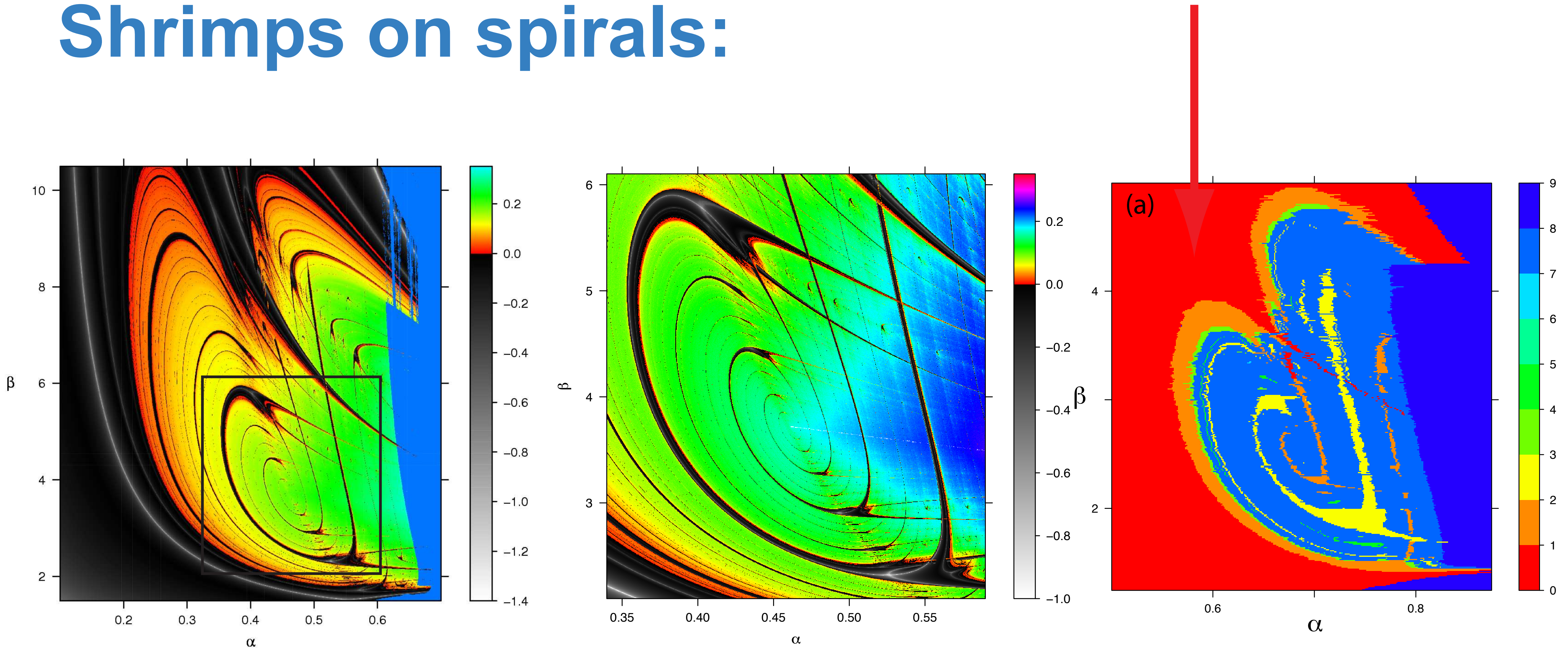


chaos

Nishio-Inaba, simulations:



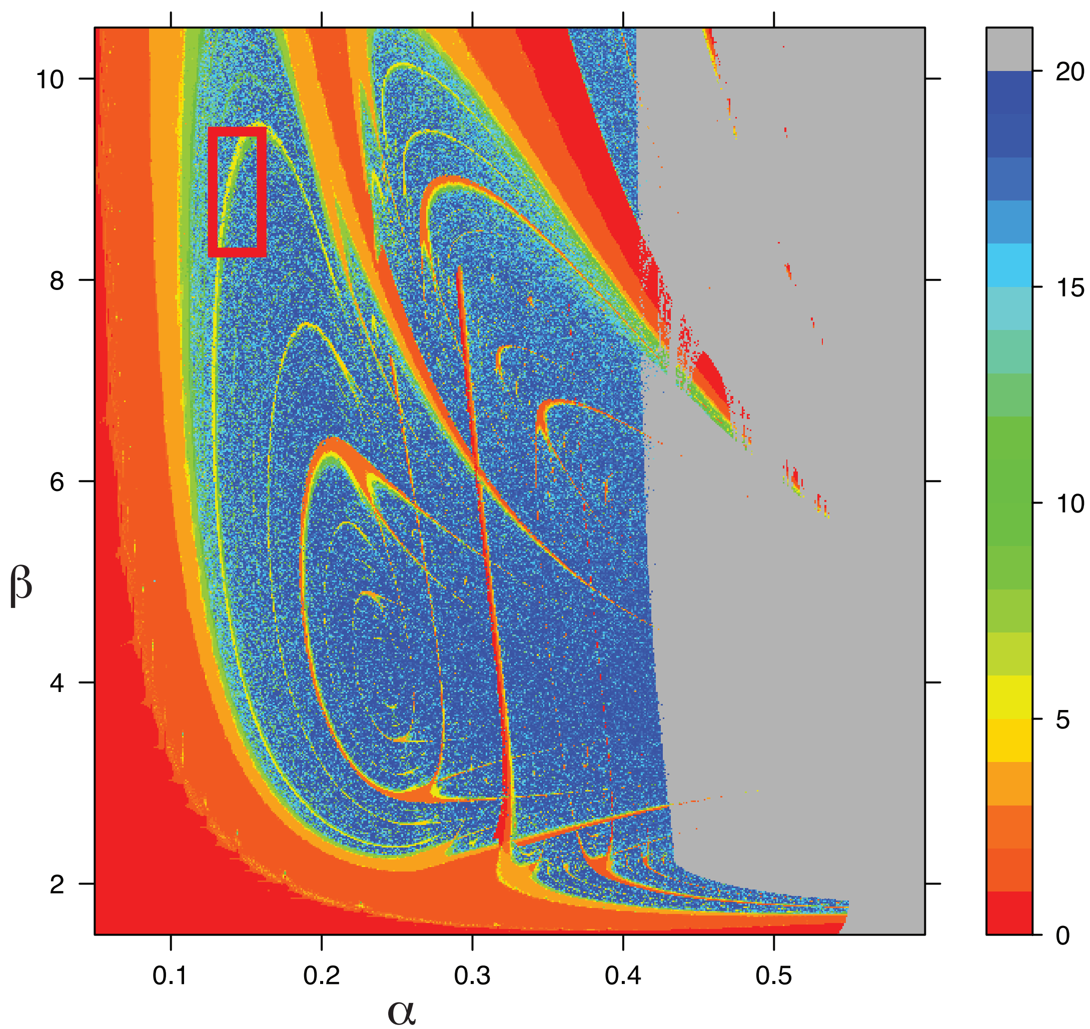
Shrimps on spirals:



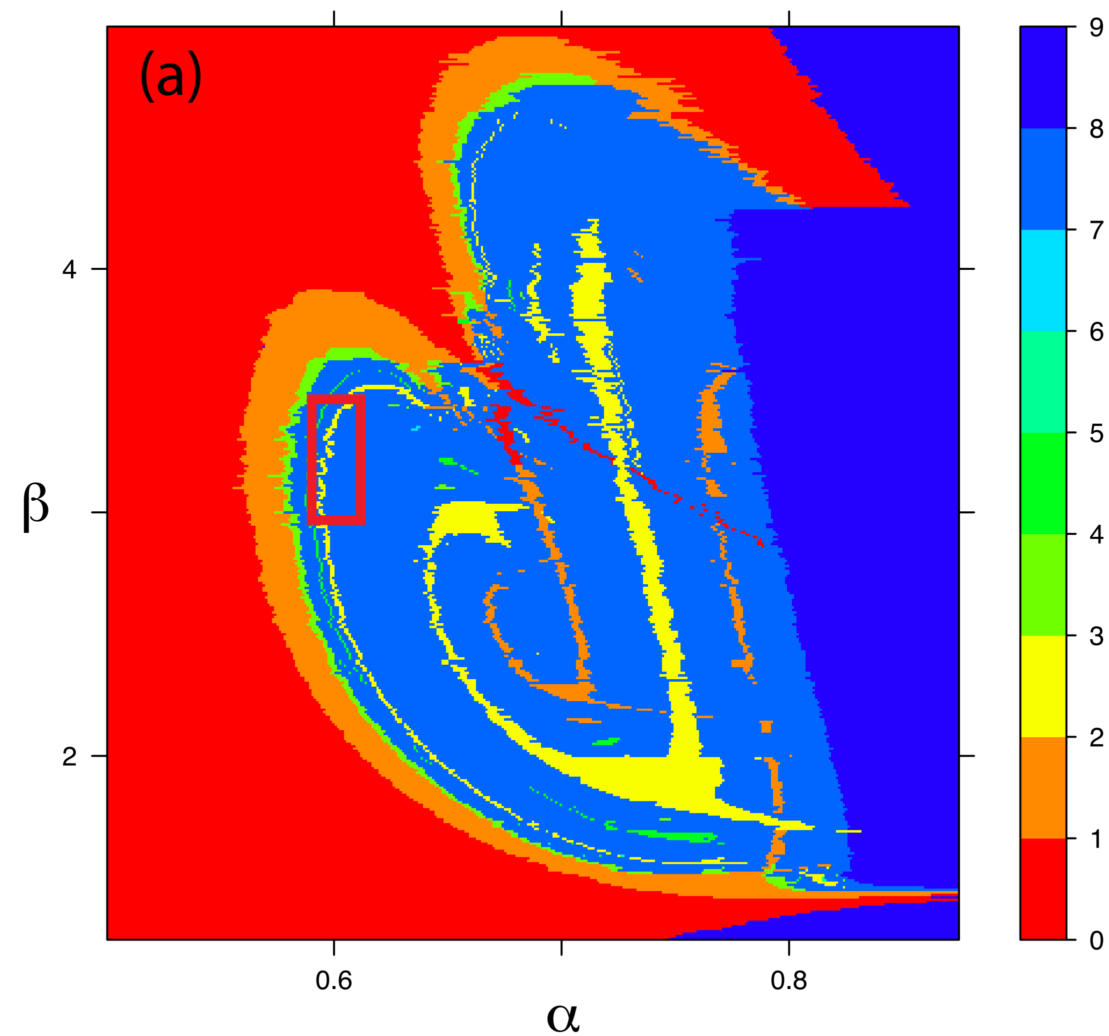
simulation, idealized, Lyapunov exponents ⟷ measurements, periodicity

metric violations!

Topological violations:



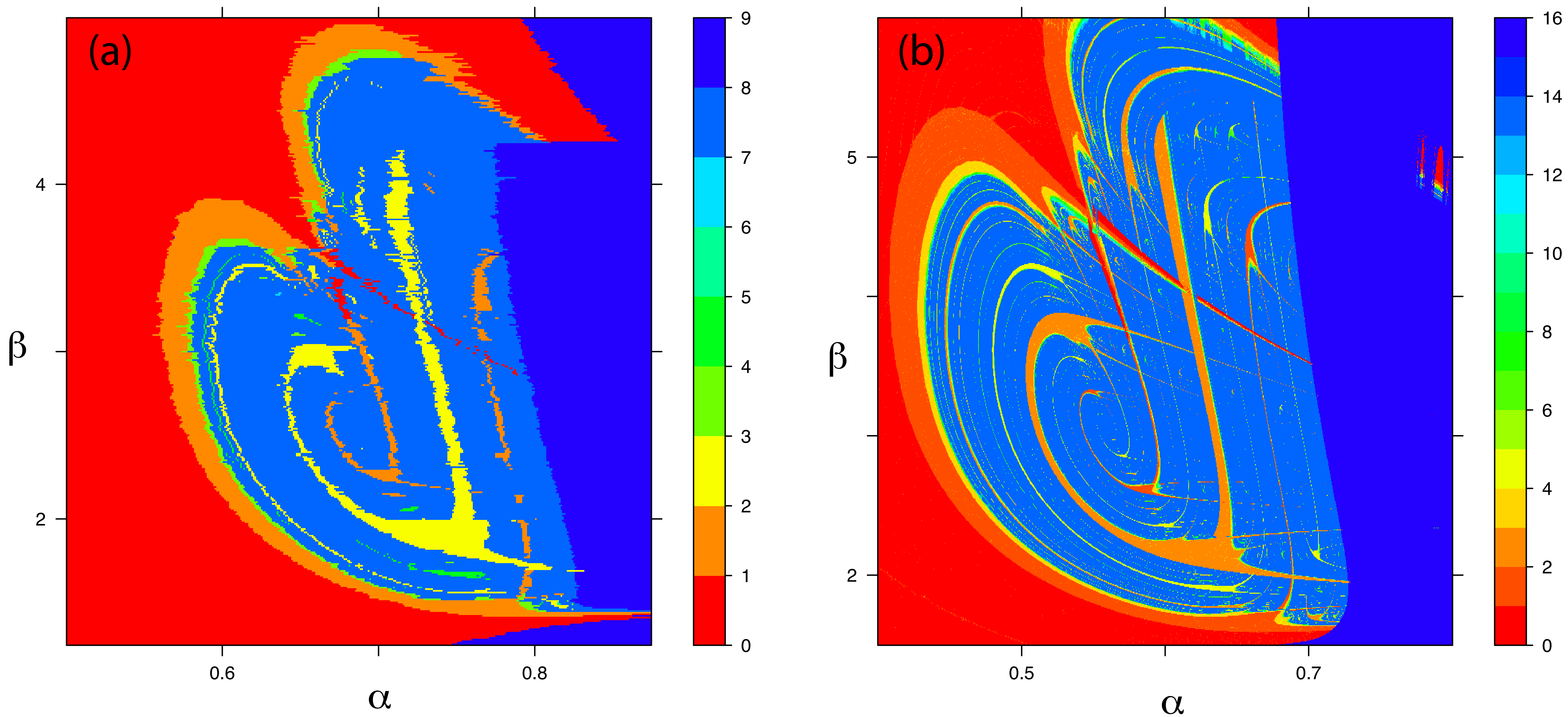
simulations



experiments

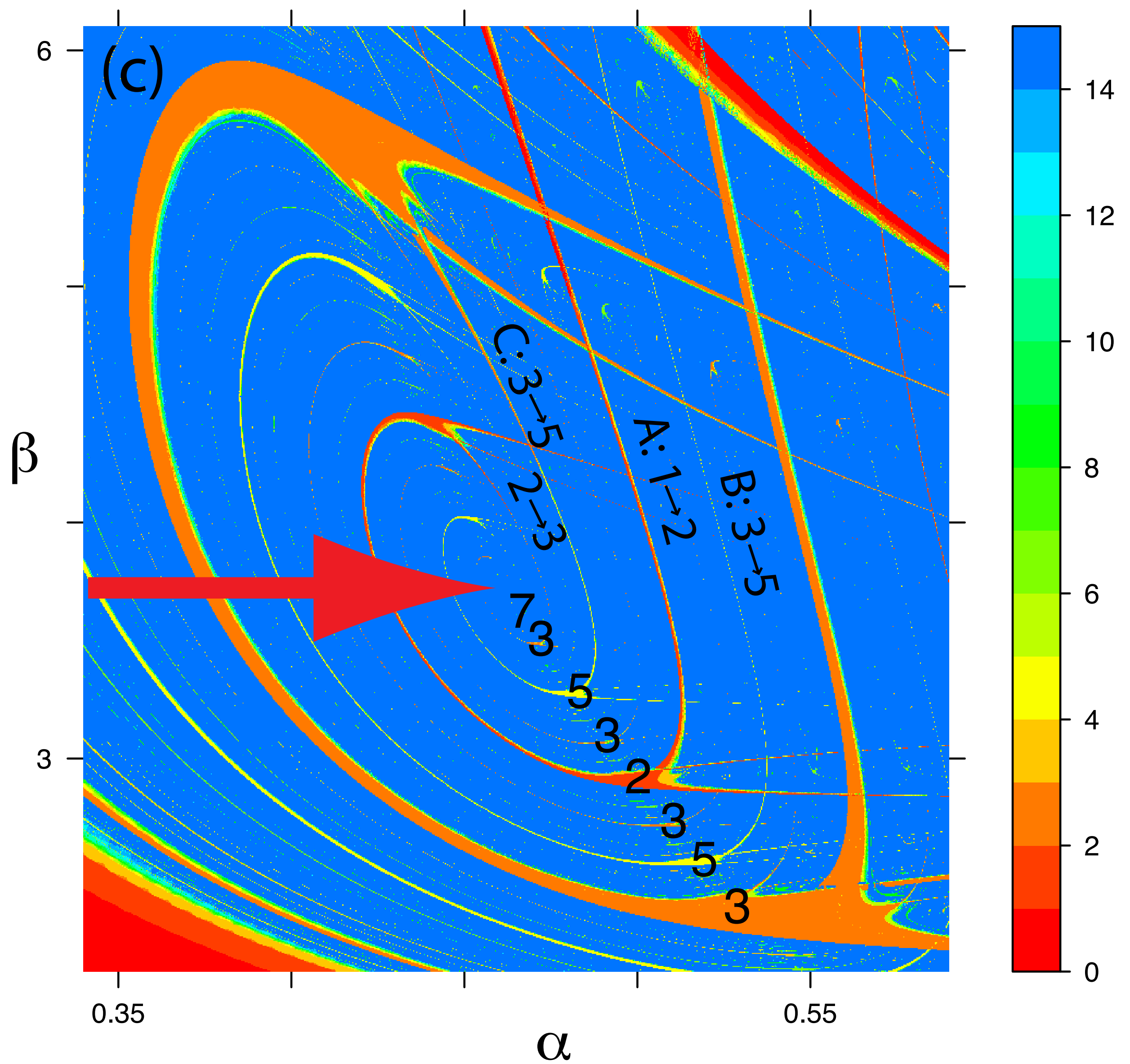
Shockley diode modeling:

$$f^{+}(y) = a\log(y/b+1),\ y \geq 0,$$
$$f^{-}(y) = -a\log(1-y/b),\ y < 0.$$



perfect agreement!

Strange ordering towards the 'hub':



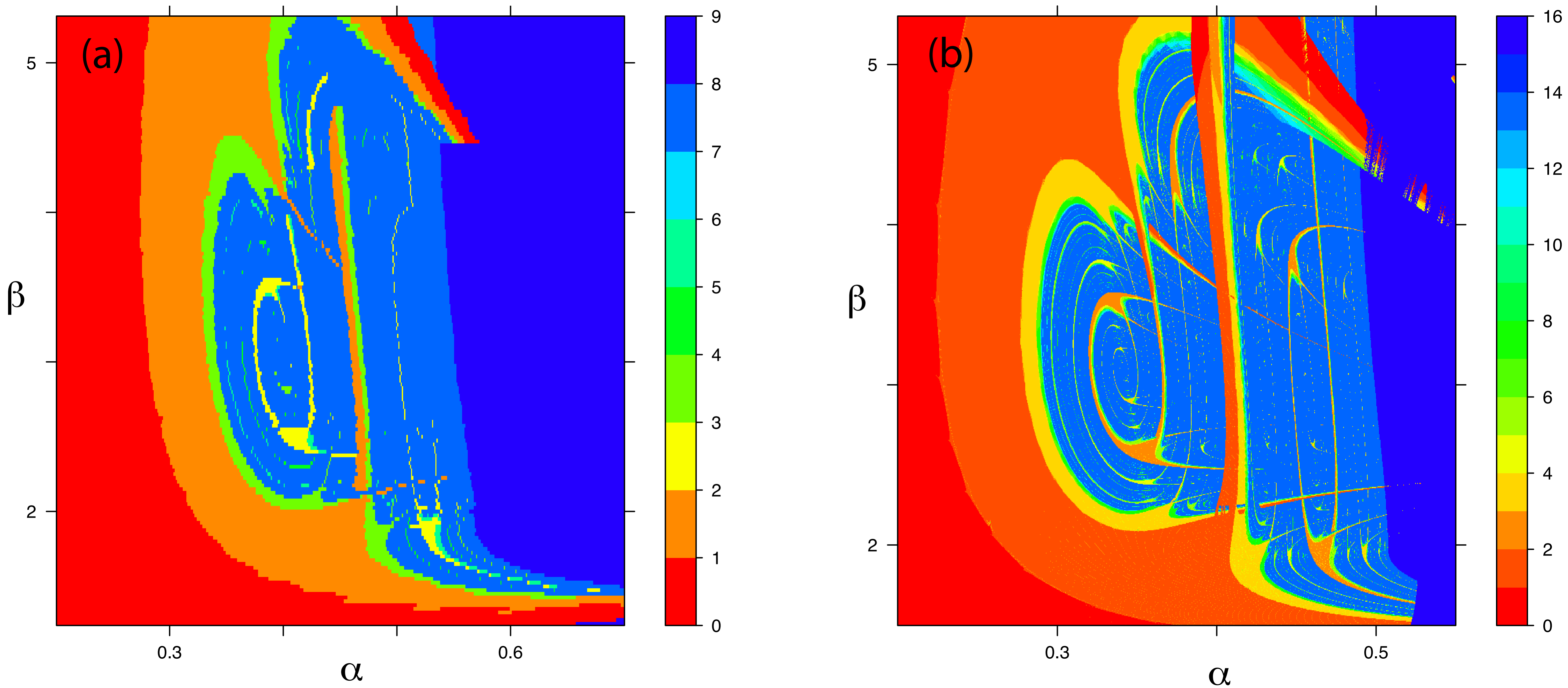
IV. Understanding ‘spirals’ and ‘hubs’:

- What is the ordering principle along the spirals?
- Is it a ‘generic’ bifurcation pattern?
- Is it a case of Shilnikov ordering?
- Experimental and theoretical evidence?
- Real-world significance?

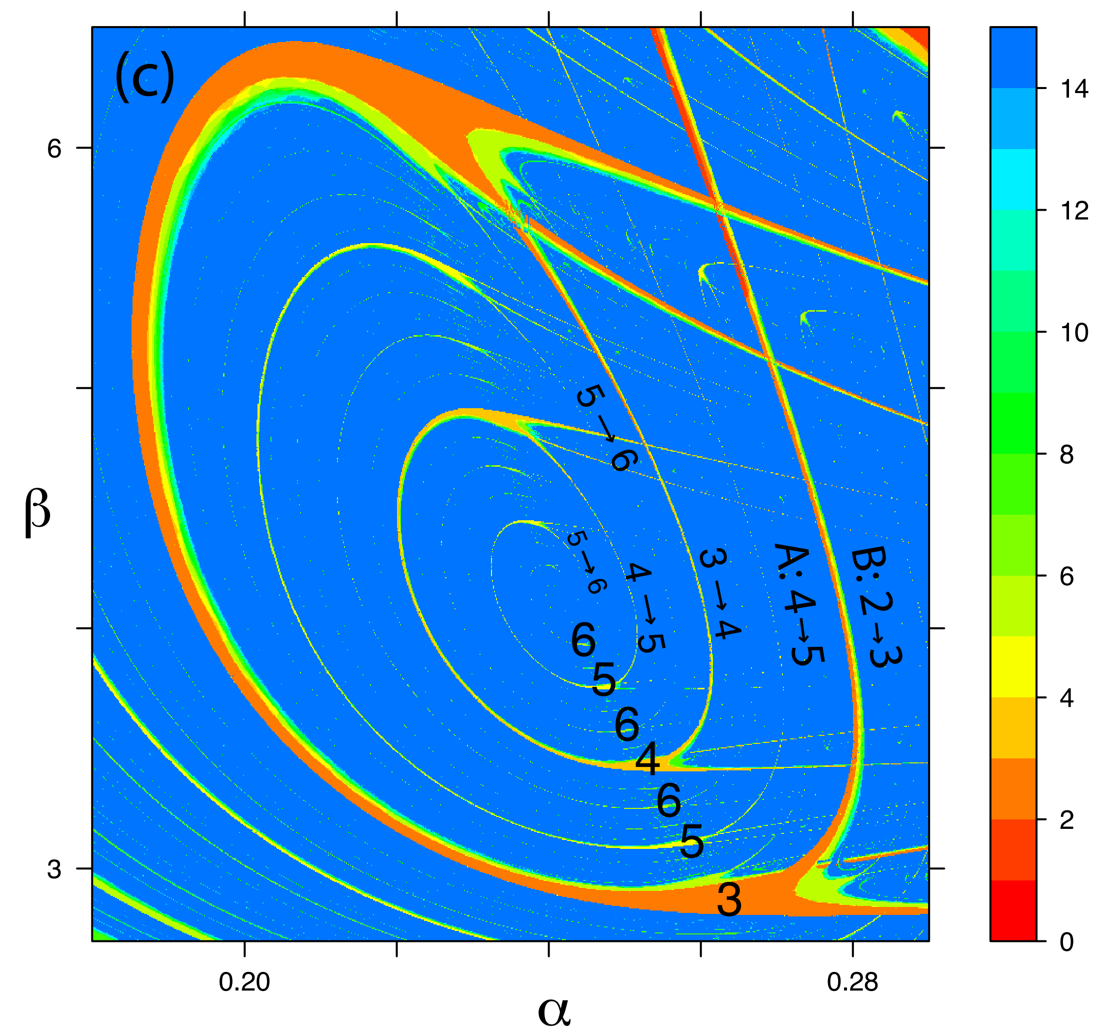
Understanding the ordering:
Simpler *asymmetric* circuit family

Bonatto-Gallas:

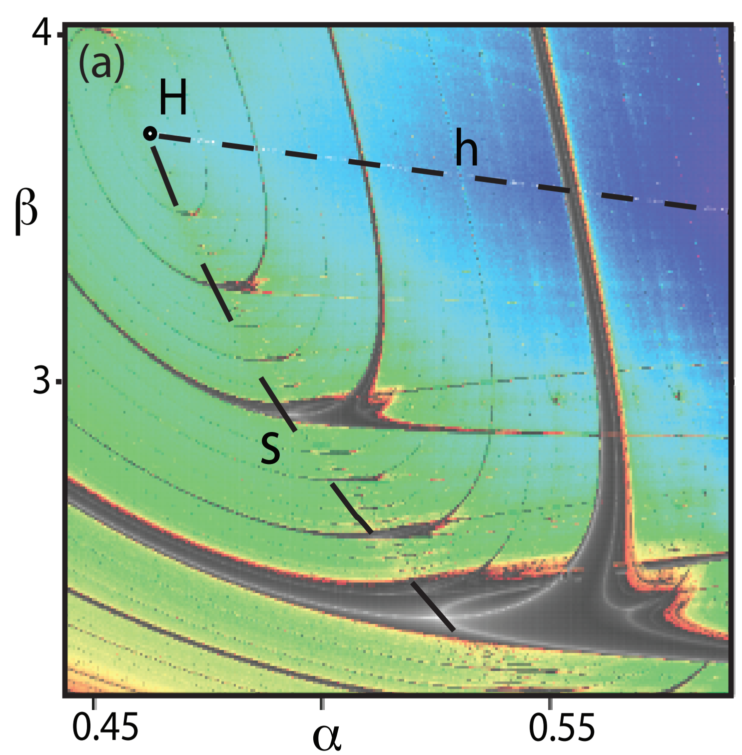
'. We selected their nice circuit because it allows spirals to be measured experimentally. In addition, uncluttered by superfluous variables and parameters, their setup produces arguably the simplest possible normal form to experimentally observe spirals and hubs.'

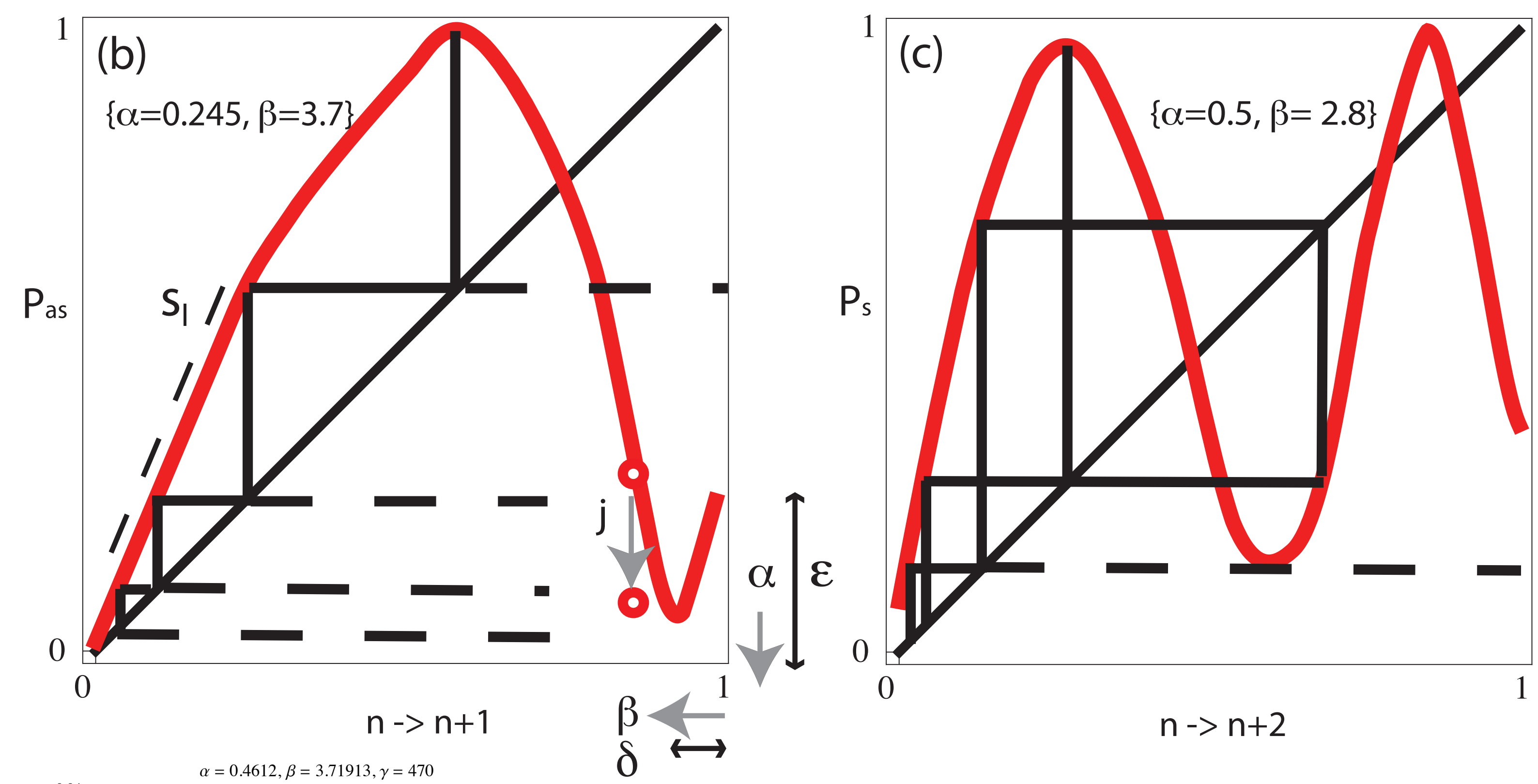


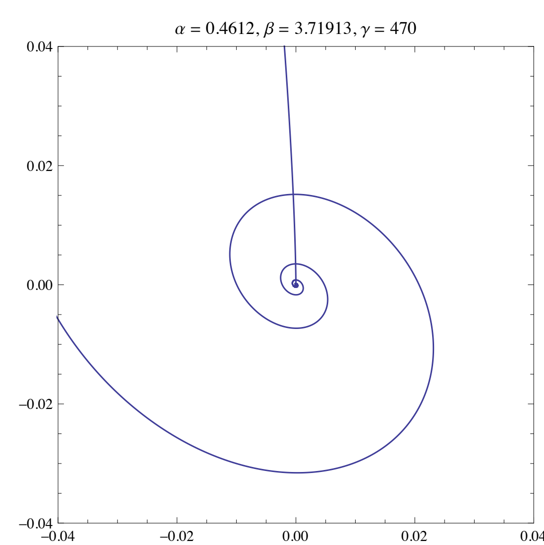
Behavior is clear for the asymmetric case:

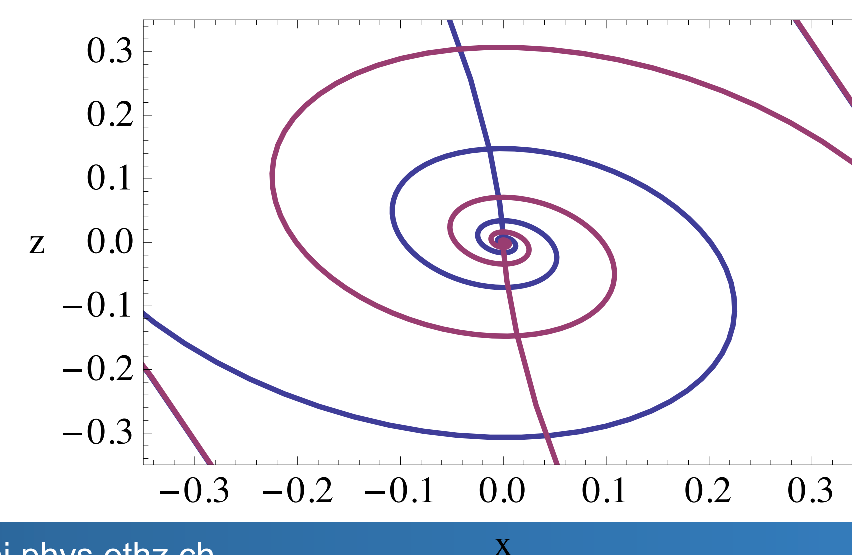


Return maps:


(a)
H
h
s
β
α
4
3
0.45
0.55


(b)
{α=0.245, β=3.7}
P_{as}
s_l
j
0
1
n -> n+1
α
ε
β
δ
(c)
{α=0.5, β= 2.8}
P_s
0
1
n -> n+2


α = 0.4612, β = 3.71913, γ = 470
0.04
0.02
0.00
−0.02
−0.04


z
x
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3

Shilnikov:

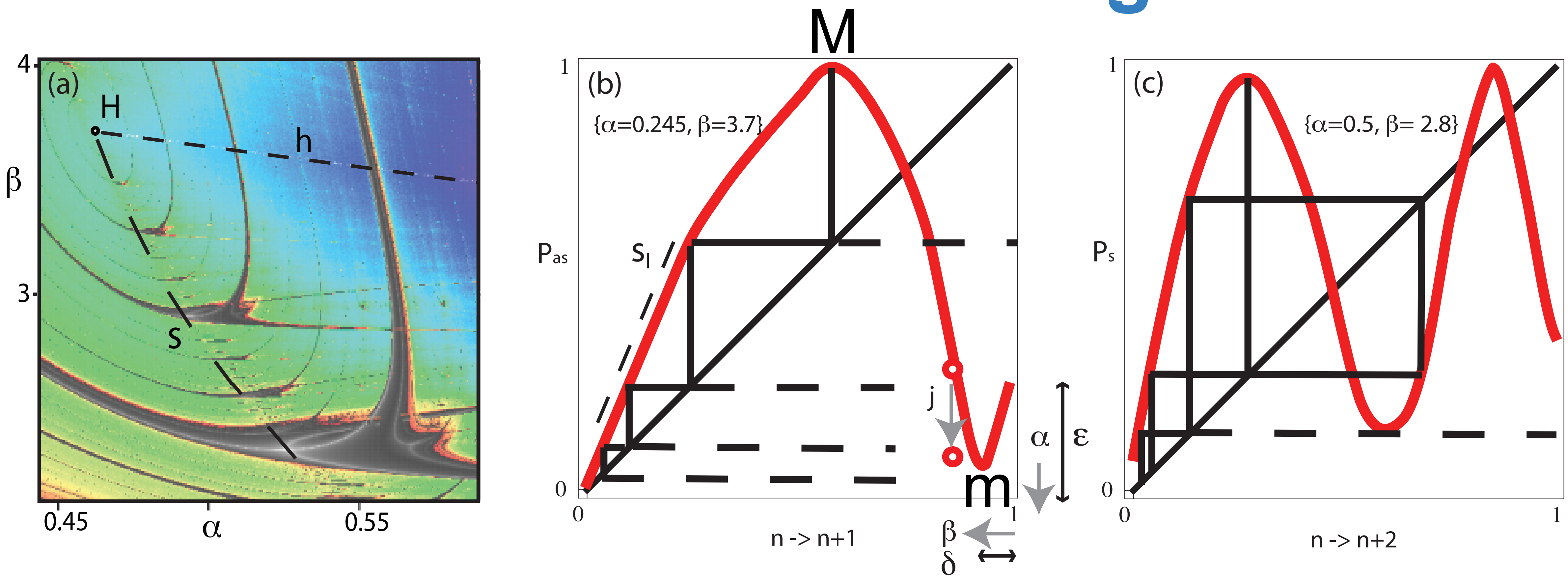
- Testing the hub for Shilnikov: conditions fulfilled. Linearization:

$$\frac{d}{dt}\vec{x} = \begin{pmatrix} \alpha & 0 & 1 \\ 0 & -f^{'}(0) & 1 \\ -1 & -\beta & 0 \end{pmatrix} \cdot \vec{x} \qquad \frac{d}{dt}\vec{x} = \begin{pmatrix} \rho & 0 & -\omega \\ 0 & \lambda & 0 \\ \omega & 0 & \rho \end{pmatrix} \cdot \vec{x}.$$

- Solution: $(\rho > 0)$

$$x(t) = e^{\rho t}(c_1 \cos(\omega t) + c_2 \sin(\omega t)),\ y(t) = e^{\lambda t},\ z(t) = e^{\rho t}(-c_2 \sin(\omega t) + c_1 \cos(\omega t)).$$

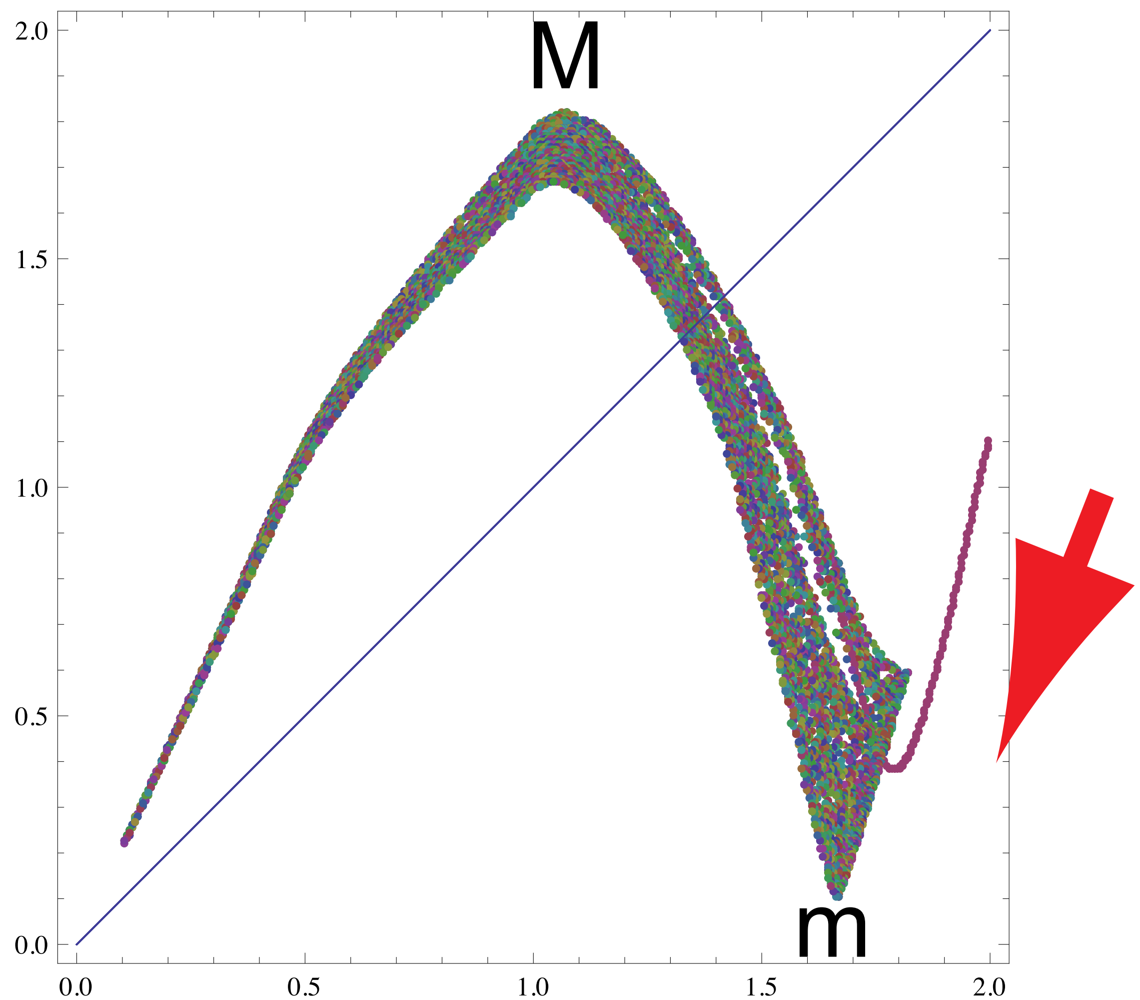
Numerical simulation scaling:



Asymmetric circuit:

- Theoretical saddle-focus scaling: $\mu_a = e^{-2\pi\frac{\rho}{\omega}} = e^{-2\pi 0.1173} = 0.48.$
- Numerical value: $(\alpha_5 - \alpha_6)/(\alpha_4 - \alpha_5) = 0.54.$
- Slope return map: $s_l \simeq 0.49^{-1}$

Family behavior:



change along line s

Return map, analytical: (Nicolis & Gaspard '84)

$$
\begin{aligned}
f_{ret,as}(x) &= s_l\, x, \;\; x\,\text{small} \\
&= 1 - c_M (x-M)^2, \;\text{around}\, M \\
&= [s_m^2 (x-m)^2 + \varepsilon^2]^{1/2}, \;\text{around}\, m.
\end{aligned}
$$

- Shrimps generated from orbits that pass through M and/or m:

$$f^n_{ret,as}(m) = 1 - c_M (s_l^{n-2}\varepsilon - M)^2 = 1 - c_M s_l^{2n-4}(\varepsilon - M s_l^{-n+2})^2 = 1 - \delta$$

$$\delta = c_M s_l^{2n-4}(\varepsilon - M s_l^{-n+2})^2$$

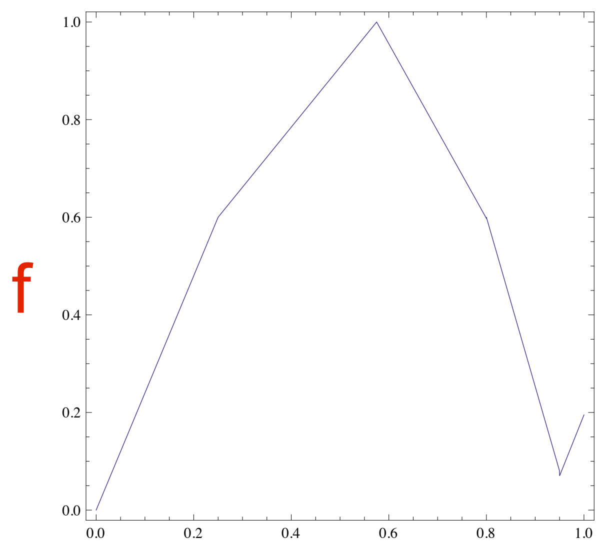
- Parabolas!

$$\varepsilon_n = M s_l^{-n+2}$$

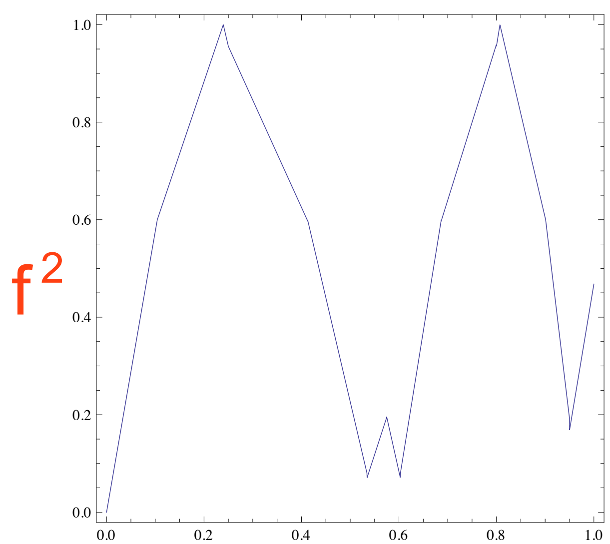
- Ellipses!

$$f^n_{ret,as}(M) = s_l^{n-2}((s_m\delta)^2 + \varepsilon^2)^{1/2}$$

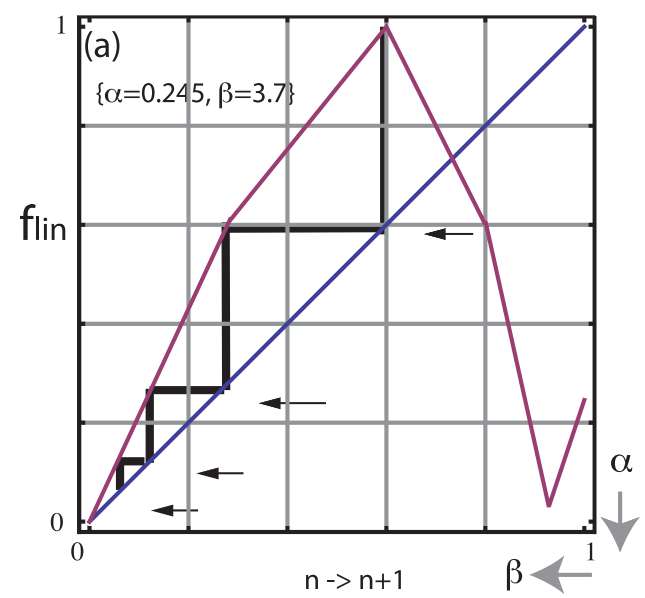
Symmetric circuit: Family affairs:



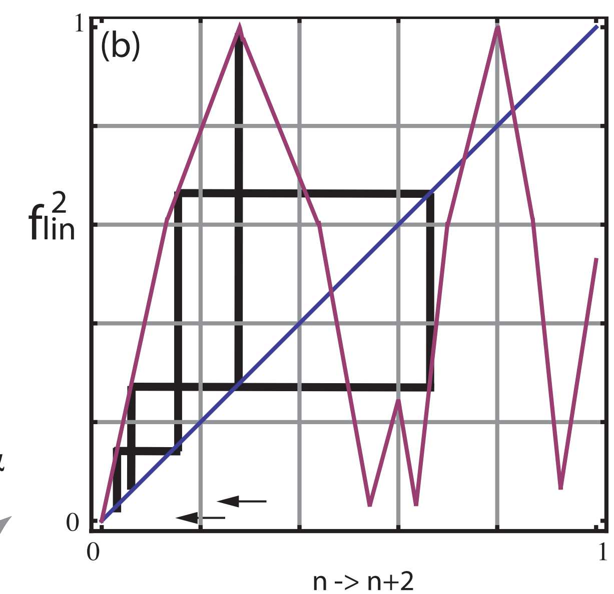
- asymmetric system



- symmetric system

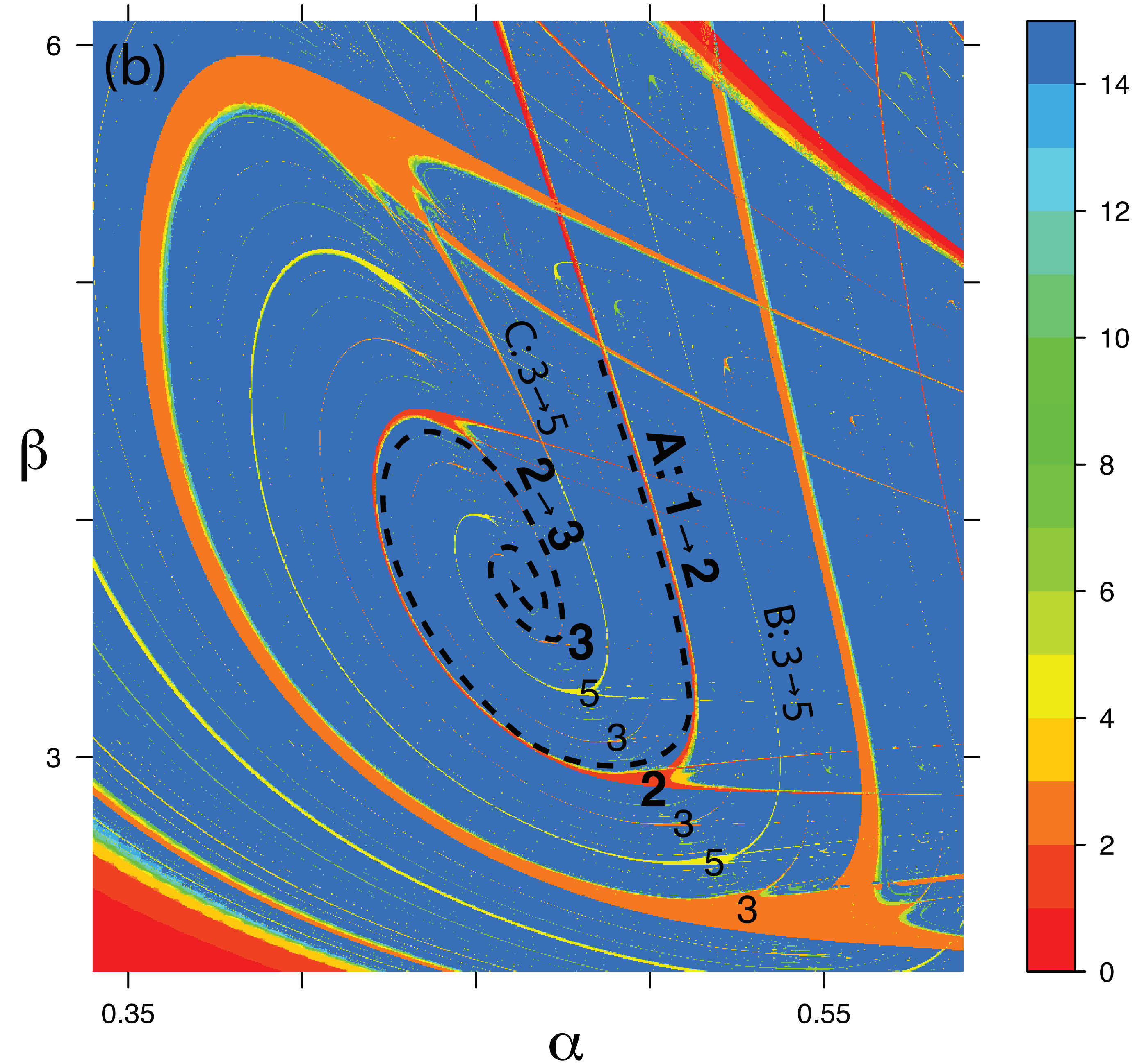


- asymmetric system



- symmetric system

Symmetric family: Asymmetric family provides a skeleton!

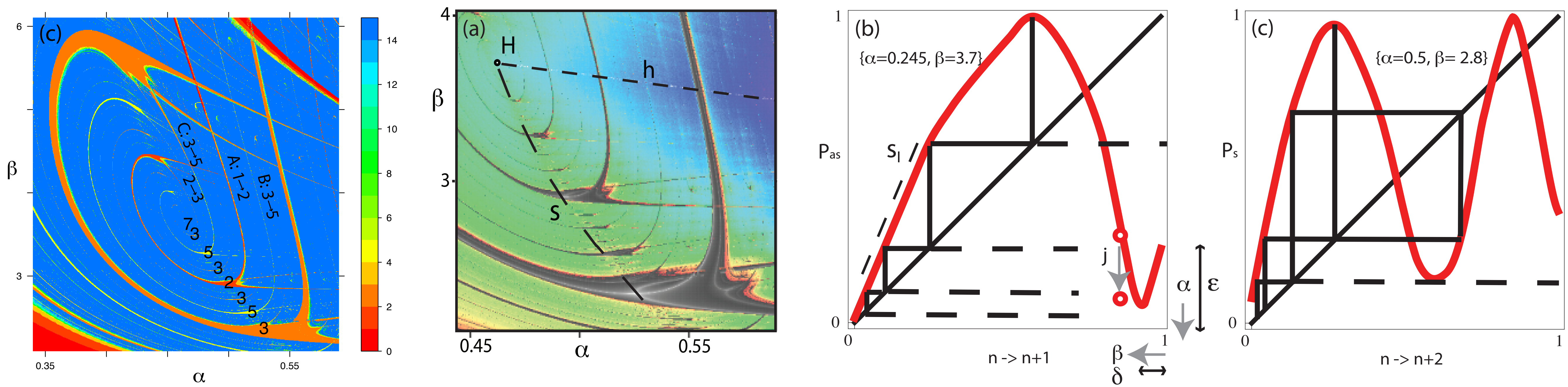


Saddle-focus scaling:

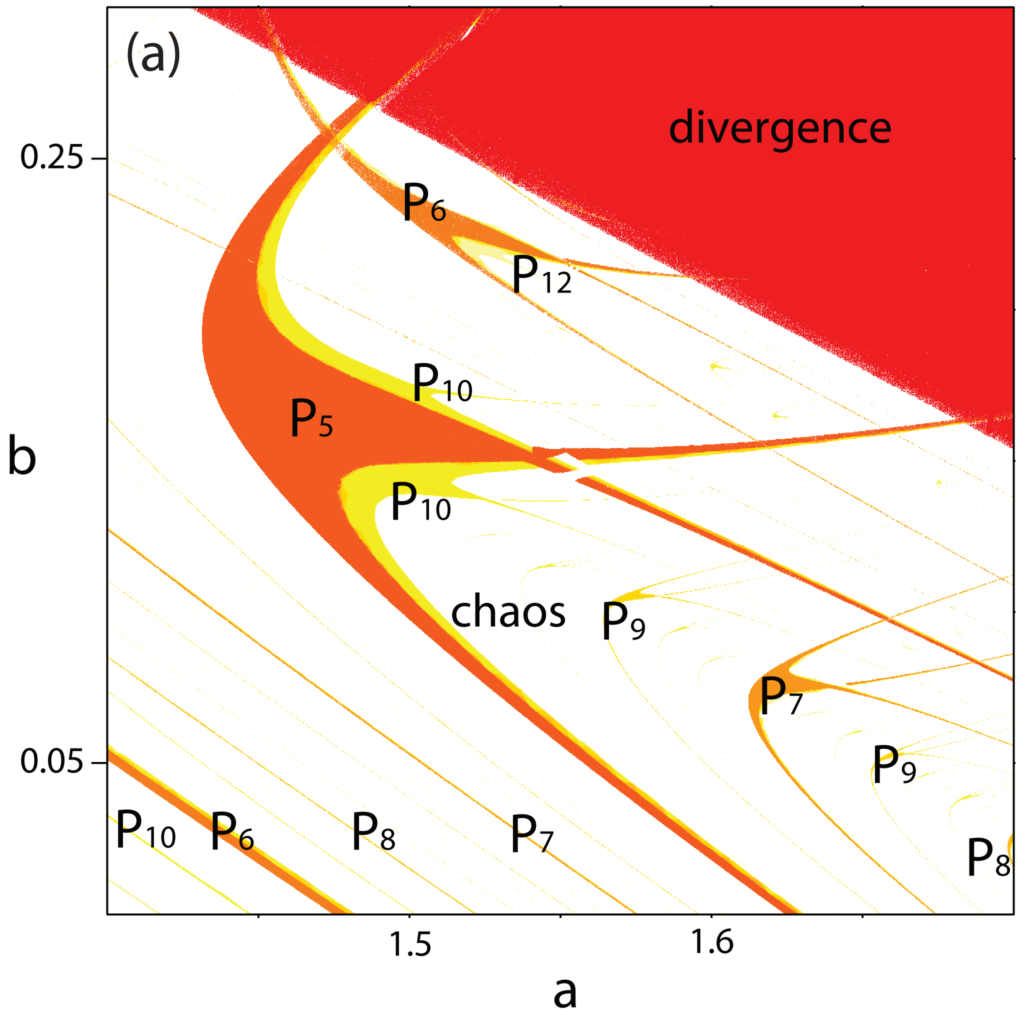
$$\mu_s = e^{-2\pi\frac{\rho}{\omega}} = e^{-2\pi 0.233} = 0.23$$

$$(\alpha_3 - \alpha_4)/(\alpha_2 - \alpha_3) = 0.25$$

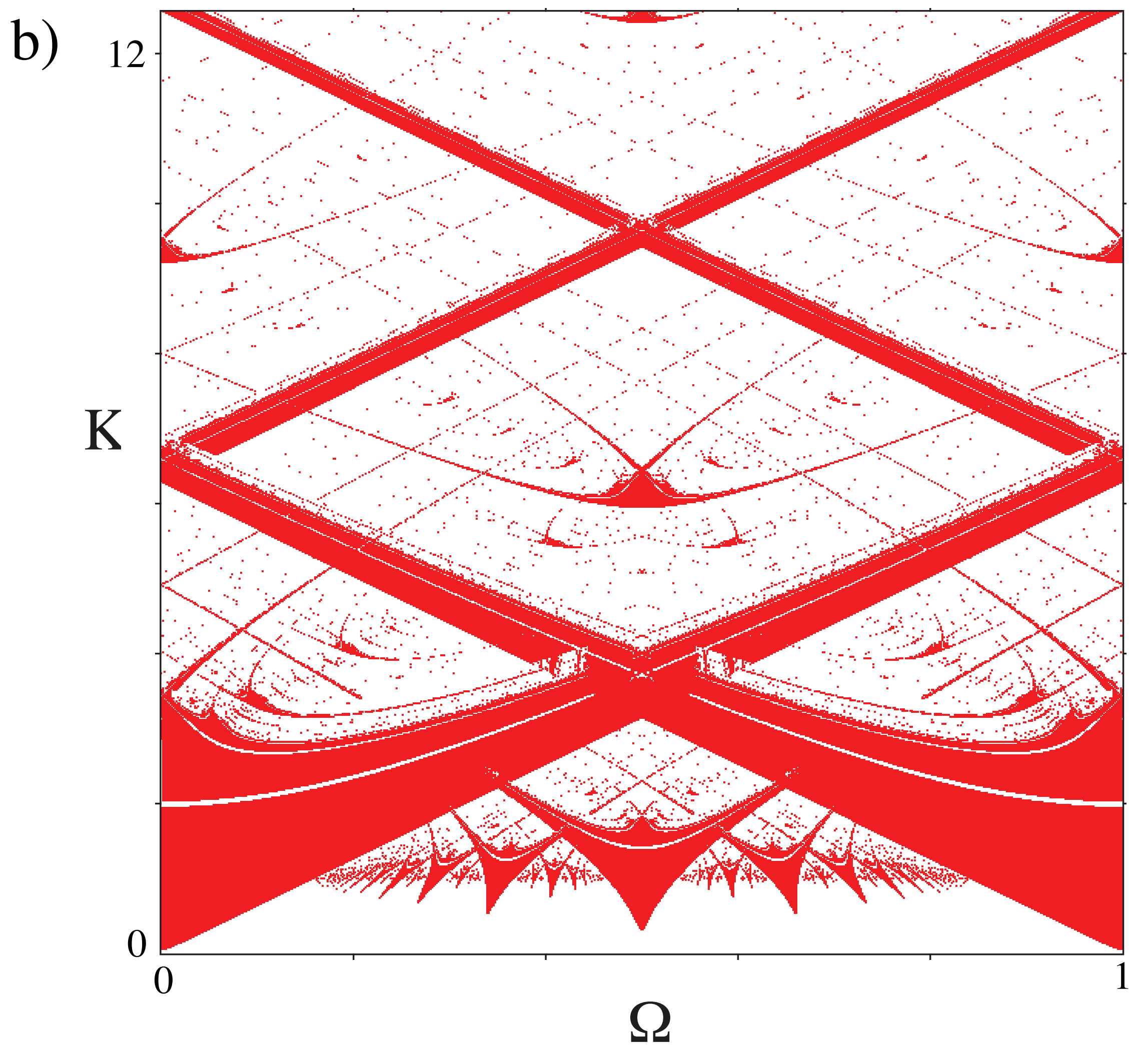
$$s_l \simeq 0.26^{-1}$$



V. Manifolds and shrimps: Back to maps

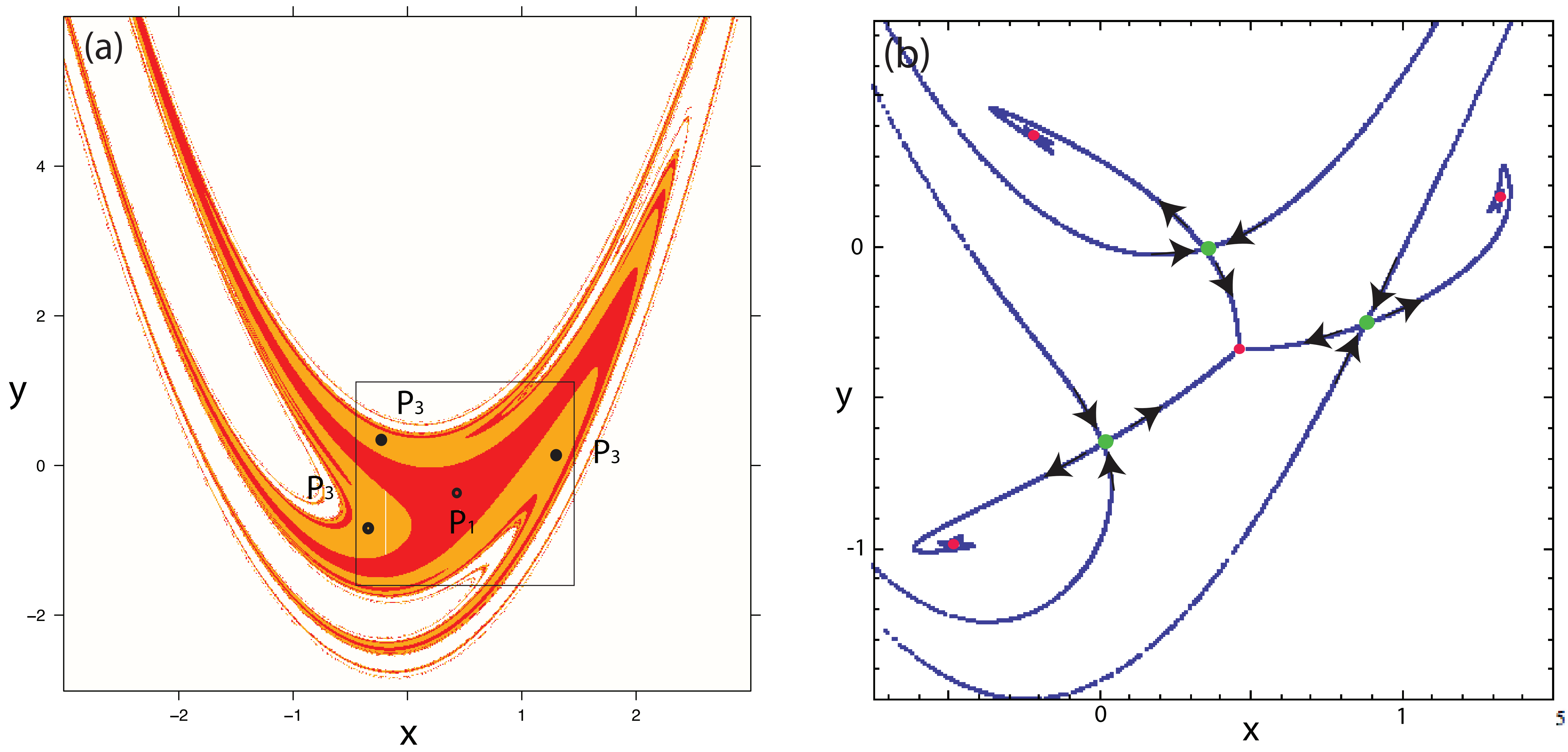


Henon



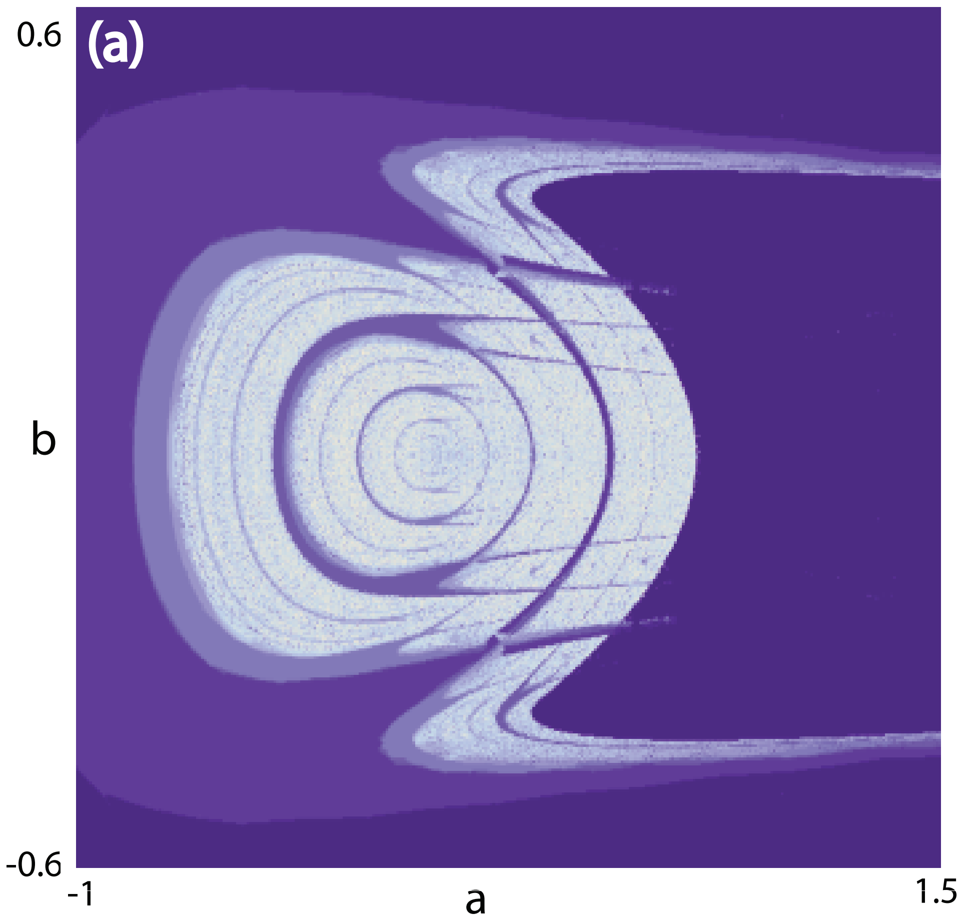
standard map

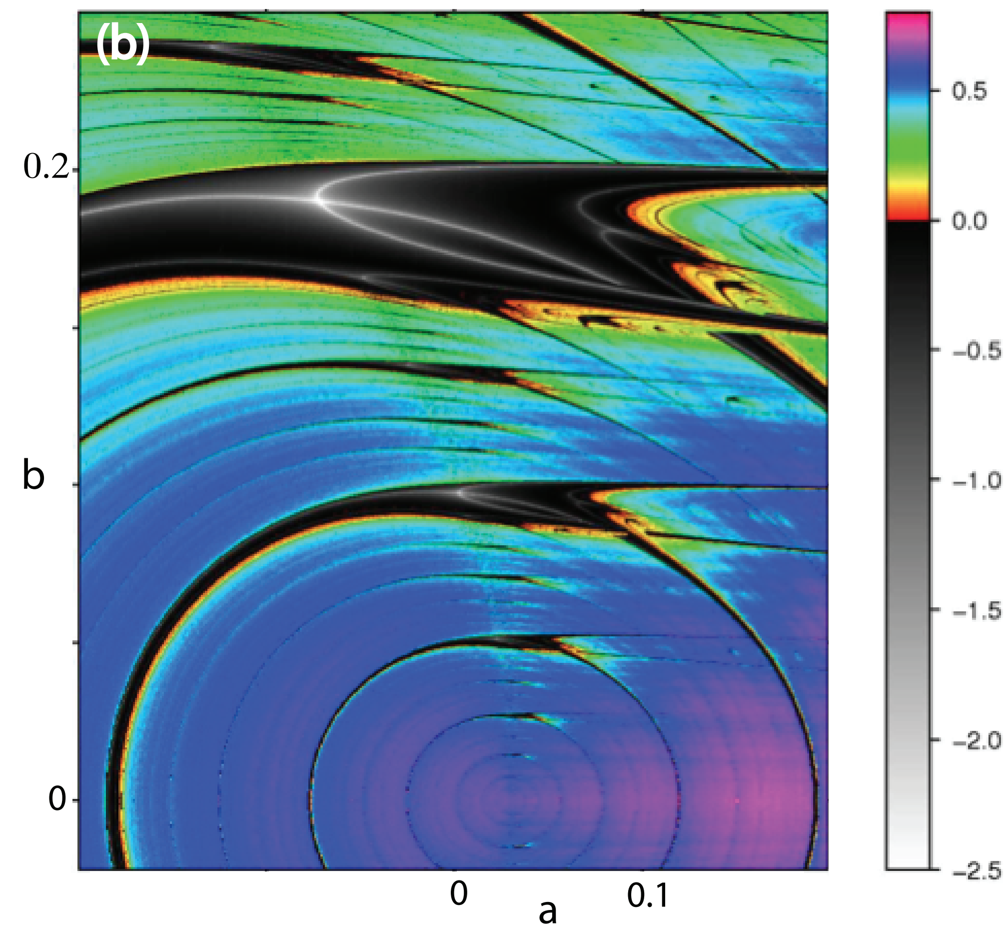
Phase-space shrimps are related to parameter shrimps!



(Henon)

Maps explain shrimps generation:



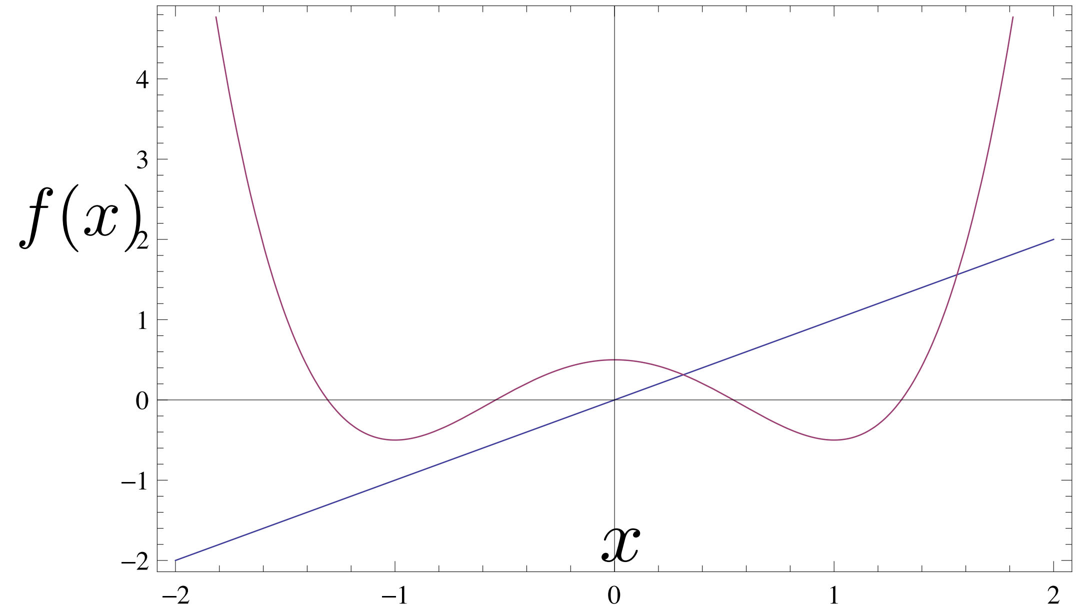


periodicities

largest Lyapunov exponent

Shrimps generation:

$$f(x) = (a - x^2)^2 - b$$ (: Gallas 1994)



iterating Henon twice to return to x!

Stable k-periodic islands then arise whenever

$$x_k = f^k(x_k), \quad | m_k | = | f^{k'}(x_k) | < 1.$$

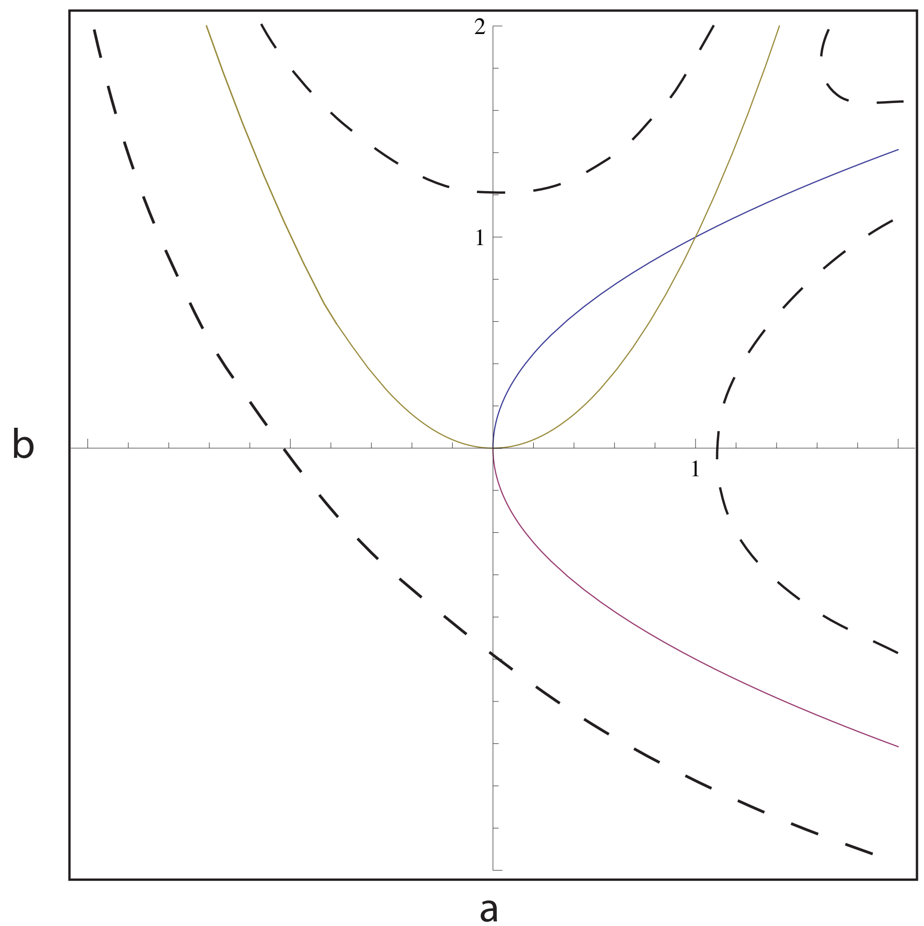
$$| m_k | = | f^{k'}(x_k) | = | -4 \prod_{i=1}^{k} x_i \prod_{i=1}^{k} (a - x_i^2) | .$$

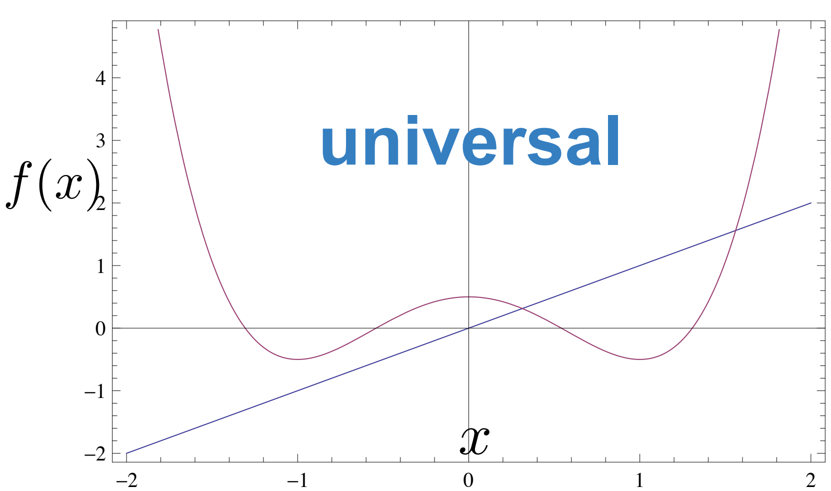
: k-superstable solutions pass either through $x_k = 0$ or $x_k = \pm\sqrt{a}$

$$x_k = 0 : \; b - (a - x^2)^2 = x \;\to\; \boxed{a = \pm\sqrt{b}.}$$

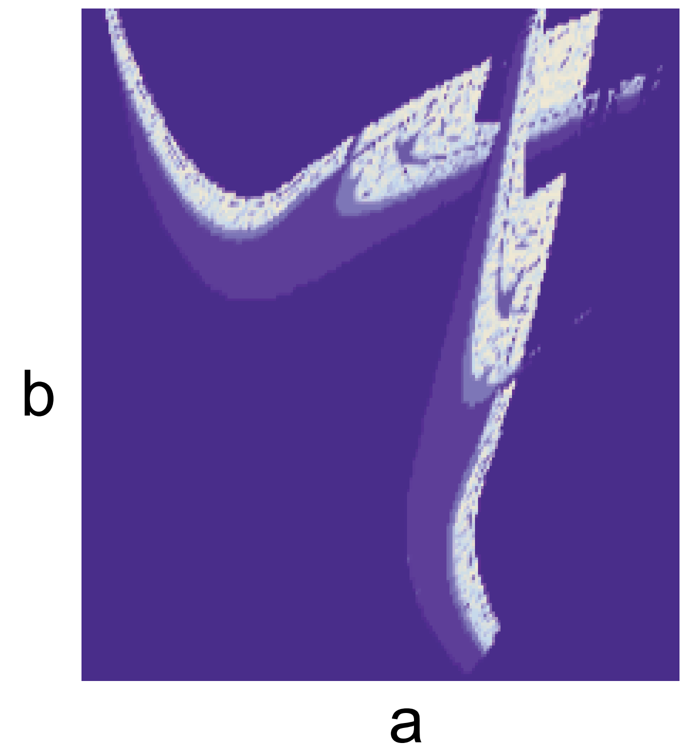
$$x_k = \pm\sqrt{a} : \;\to\; \boxed{b = \pm\sqrt{a}.}$$

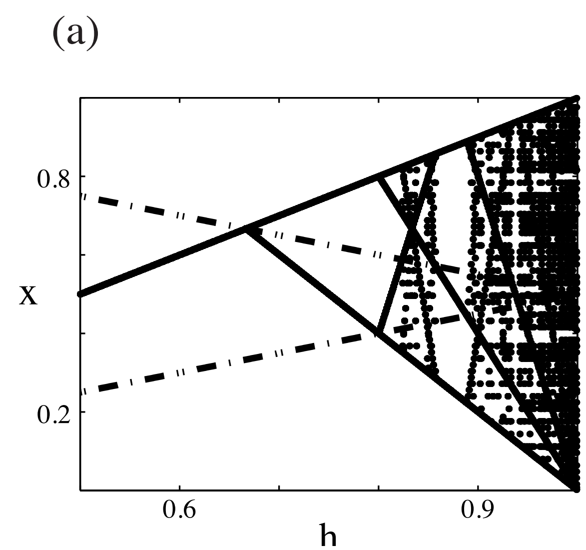
Shrimps generation

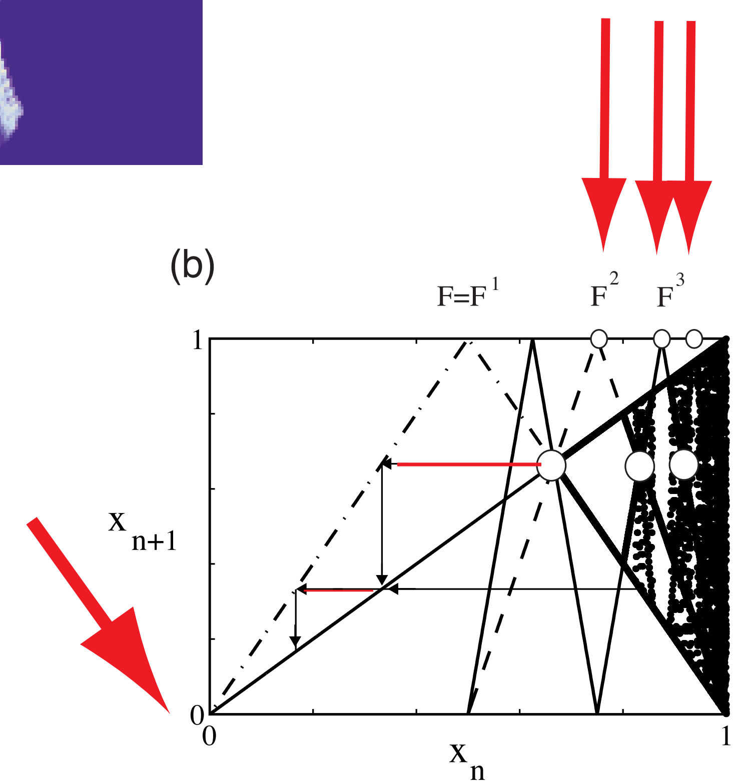




Shrimps line-up







non-universal

Shrimps line-up: (for Gallas map 1994)

- k=1-periodic islands: $$a^2 - b - x - 2ax^2 + x^4 = 0$$ $$-m - 4ax + 4x^3 = 0.$$

- Phase-space variable x can be eliminated and the problem can be examined in the {a,b}-parameter space, leading to

$$m^4 - 12m^3 + (48 - 32ab)m^2 + 64(ab - 1)m - 256(a - b^2)(a^2 - b) = 0$$

- Conditions for the family that passes through x=0 / $x = \pm\sqrt{a}$:
- k=1: $a^2 - b = 0$ $\quad$ $-b = 0$
- k=2: $(a - (a^2 - b)^2)^2 - b = 0$ $\quad$ $(a - b^2)^2 - b = 0$
- k=3: $(a - (a - (a^2 - b)^2)^2 - b = 0$ $\quad$ $(a - ((a - b^2)^2 - b)^2)^2 - b = 0$

- Intersections: Pairs of equations (Gallas)

$$a = \pm\sqrt{b \pm \sqrt{a \pm \sqrt{b \pm \ldots}}},$$
$$b = \pm\sqrt{a \pm \sqrt{b \pm \sqrt{a \pm \ldots}}}$$

b

a

- Incorrect fixed point scaling

- Lacks jump discontinuity system (dimensionality)

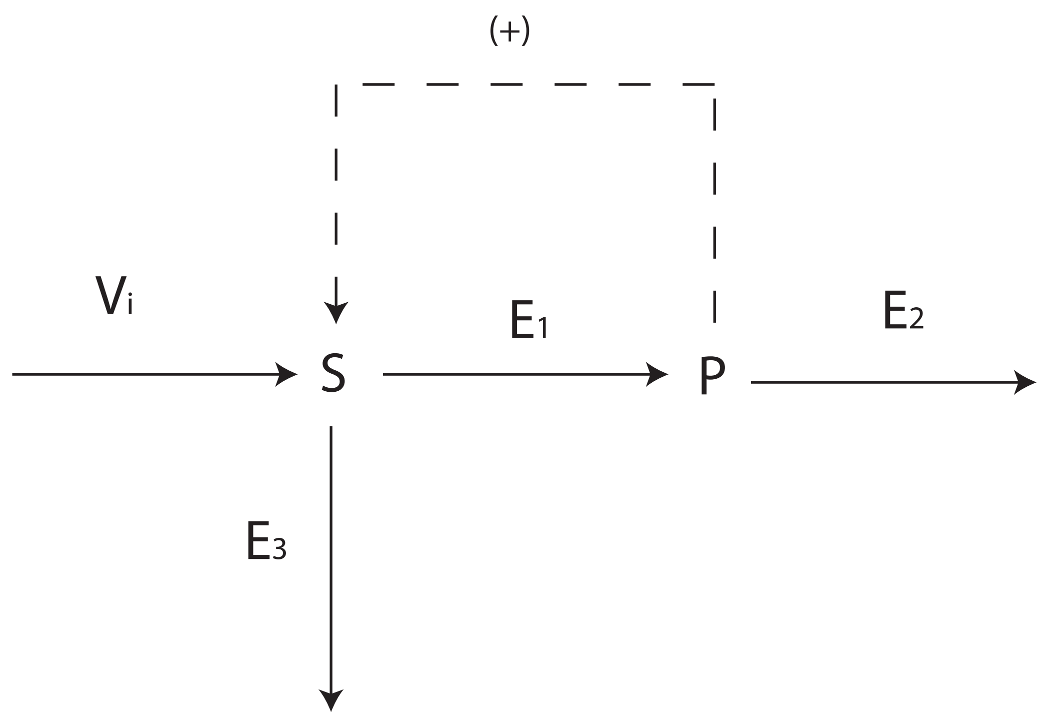
No spirals!

VI. Shrimps relevance: Systems Biology:

- Data points: Express states, substances, organisms
- Geometric boundaries of data clouds assumed to be Gaussian
- Set of parameters allowing for biochemical reaction processes needed
- Particular interest: Time-dependence, e.g. periodicity
- Interest: Set of variables or parameters that lead to identical regular temporal behavior, i.e., periodic solutions

Example: (Goldbeter 1984)

- Positively regulated chemical reaction process of two molecular species:



- Standard method: Conversion into a system of - generally nonlinear - differential equations:

Equations:

- Michaelis - Menten approximation of the mass-balance equation:

$$\frac{d[S]}{dt} = \boxed{V_i} - \frac{V_{E_1}^{max}[S]/k_S(1.+[S]/k_S)(1.+[P]/k_P)^2}{L+(1.+[S]/k_S)^2(1.+[P]/k_P)^2} - \frac{V_{E_3}^{max}[S]}{k_{m3}+[S]}$$

$$\frac{d[P]}{dt} = \frac{V_{E_1}^{max}[S]/k_S(1.+[S]/k_S)(1.+[P]/k_P)^2}{L+(1.+[S]/k_S)^2(1.+[P]/k_P)^2} - \frac{V_{E_2}^{max}[P]}{k_{m2}+[P]}$$

where L, and for all subscripts, $V_{\{..\}}^{max}$ and $k_{\{..\}}$ are parameters and [ ] denotes chemical concentration.

Generated objects and behaviors:

- In dependence of Vi:

  Systems display complex bifurcation transitions typical for nonlinear systems

  (Eiswirth 2005)

- Practical task: In gene expression: m measurements of n genes define a set of n points in an m-dimensional space:

We search for subdomains that correspond to different biological states of the regulatory system. This is done by means of clustering.

(Belkin and Niyogi (2003), Donoho and Grimes (2003), Weinberger (2004))

Shrimps prohibit classical clustering!

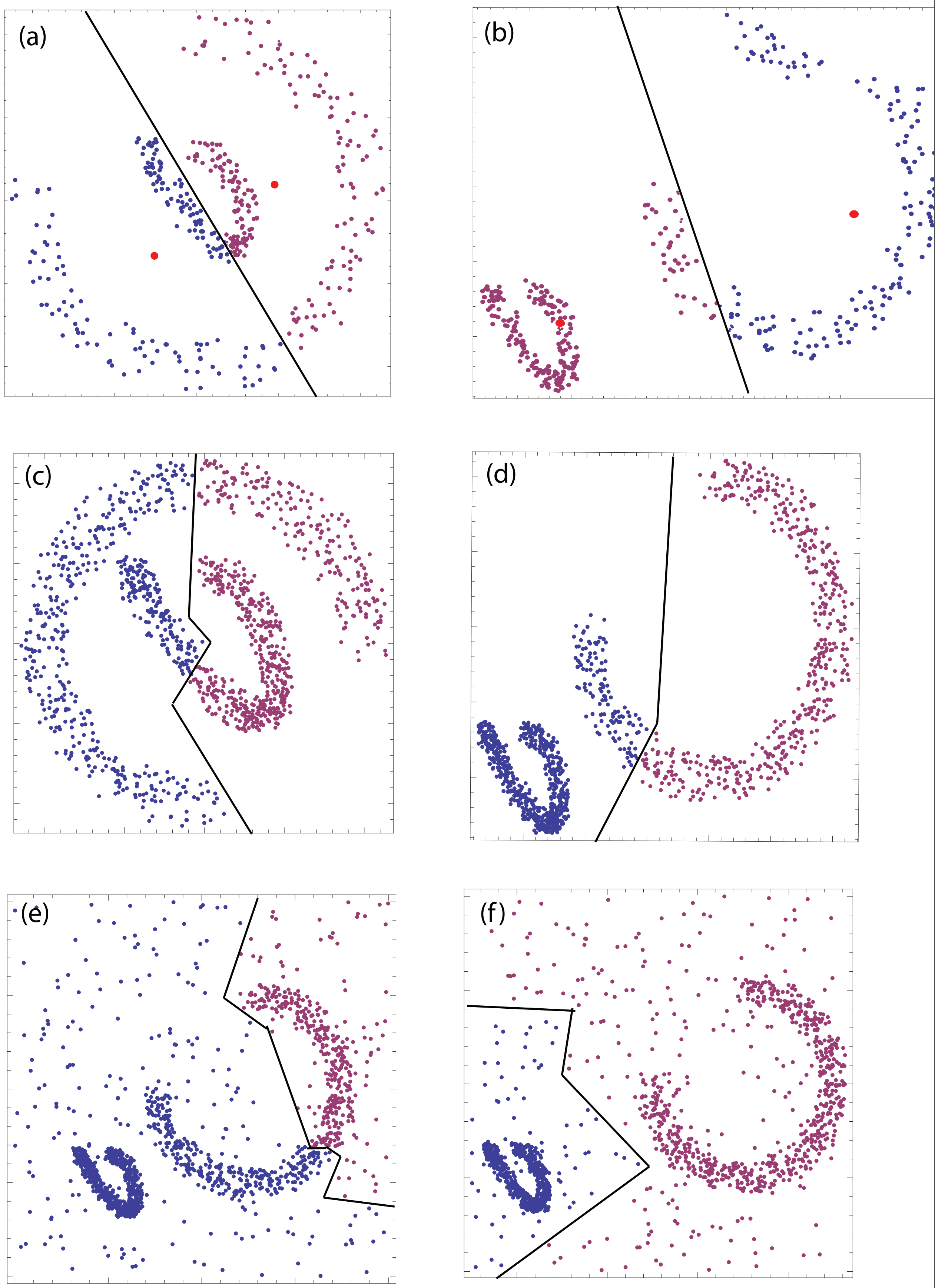
- Clustering objects:

  convex-concave sets w / o

  background noise

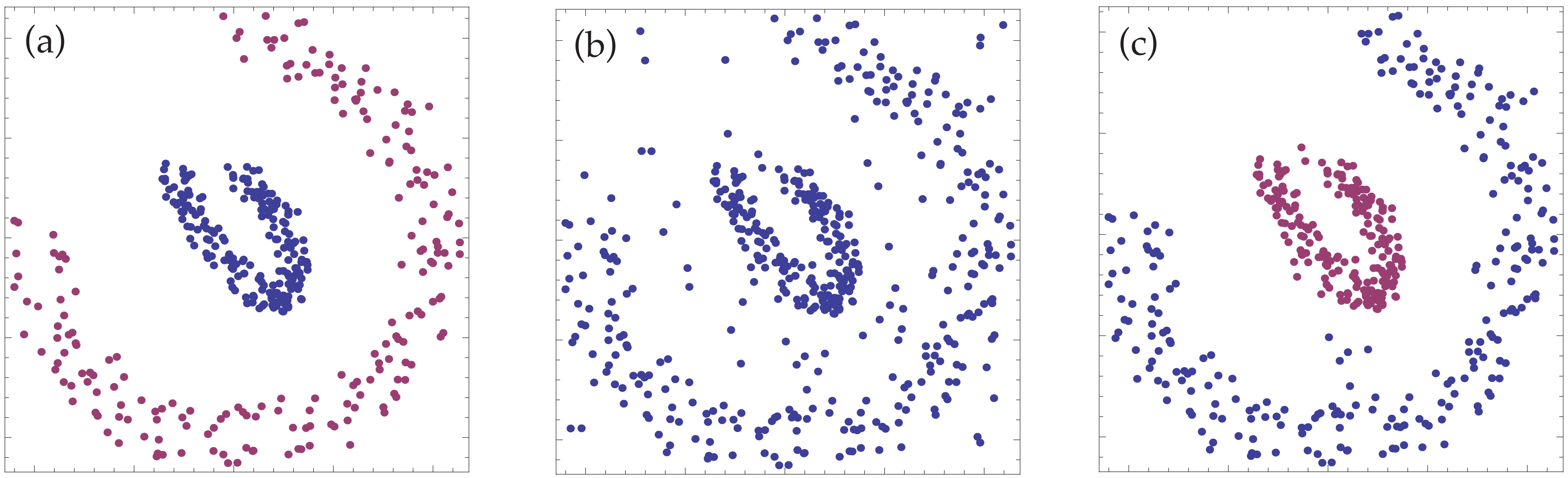
- k-means and Ward's

  clustering approach fail:



Remedy:

Use fully local, spin- or neural network-like systems!



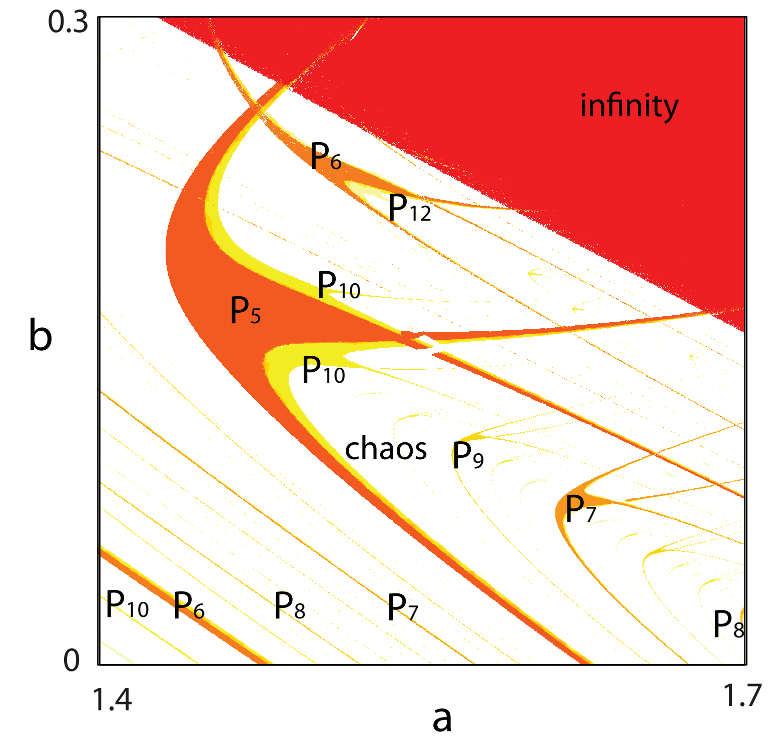
Reason for difference in performance:

k-means / Ward use non-local or semi-local neighborhood notions:

k-means: distance from central point

Ward's clustering: distance from a point to a cluster : supervised

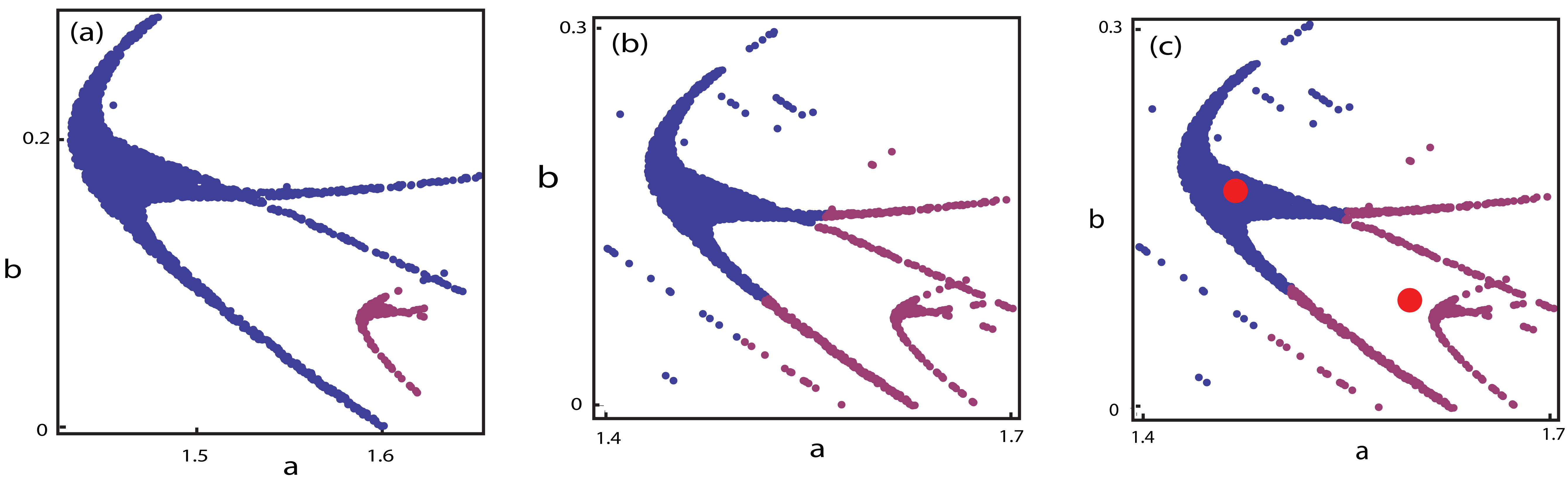
Shrimps clustered:



HLC: success

Ward: failure

k-means: failure



Dienstag, 26. Juli 2011

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- Real-World Existence and Origins of the Spiral Organization of Shrimp-Shaped Domains, R.Stoop, P. Benner, and Y. Uwate, PRL 105, 074102, 2010 (cover)
- Shrimps: Occurrence, scaling and relevance, R. Stoop, S. Martignoli, P. Benner, R. L. Stoop, and Y. Uwate, IJBC, in press
- Clustering, bioinformatics:
- Hearing research, pitch, speech processing:
- Biophysics & electronic Hopf Cochlea:
- Networks, Drosophila, ..

} www.ini.uzh.stoop

~~Shrimps~~ now!

beer

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