

Image Edge Detection with Discretely Spaced FitzHugh- Nagumo Type Excitable Elements

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Outline

- Introduction & Background
 - Reaction-diffusion, coupled oscillators & pattern formation
 - Edge detection algorithms
- Motivation
- Edge Detection Algorithms
 - Our previous algorithm utilising coupled excitable elements
 - Proposed algorithm based on the previous algorithm
- Experimental Results
- Conclusion

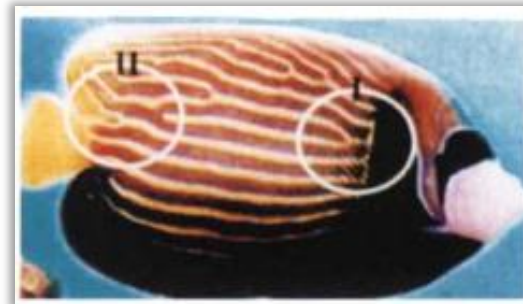
Introduction & Background:

Reaction-Diffusion, Coupled Oscillators & Pattern Formation

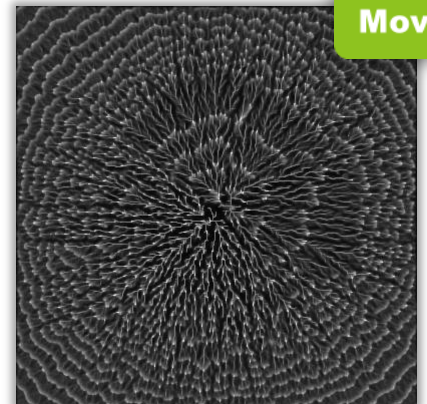
- Reaction-Diffusion System
 - activator and inhibitor
- Weak Inhibition
 - Chemical reaction
 - Belousov-Zhabotinsky (BZ) reaction
 - Biological system
 - signal propagation in *Dictyostelium discoideum*
- Strong Inhibition
 - Turing pattern



Keener & Tyson, *Physica D*, 1986



Kondo & Asai, *Nature*, 1995



Höfer et al., *Physica D*, 1995

Introduction & Background:

Previous Research Topics on Pattern Formation

	Weak Inhibition	Strong Inhibition
Experimental Study	<p>Belousov-Zhabotinsky (BZ) Reaction - Zaikin & Zhabotinsky, <i>Nature</i>, 1970</p> <p>Light-Sensitive BZ Reaction - Busse & Hess, <i>Nature</i>, 1973 - Jinguji et al., <i>Physica D</i>, 1995</p>	<p>Mach Bands in <i>Limulus</i> Eyes - Hartline et al., <i>J. Gen. Physiol.</i>, 1956</p> <p>Chemical Reaction - Castets et al., <i>Phys. Rev. Lett.</i>, 1990</p> <p>Pattern Formation on Fish Skin - Kondo & Asai, <i>Nature</i>, 1995</p>
Modelling Study	<p>FitzHugh-Nagumo (FHN) Model - FitzHugh, <i>Biophysical J.</i>, 1961 - Nagumo et al., <i>Proc. IRE</i>, 1962</p> <p>Reaction-Diffusion + Oregonator (BZ) - Keener & Tyson: <i>Physica D</i>, 1986</p>	<p>Pattern Formation in <i>Hydra</i> - Turing, <i>Proc. Roy. Soc. Lond.</i>, 1952 - Gierer & Meinhardt, <i>Kybernetik</i>, 1972</p> <p>Model for Mach Bands - Barlow & Quarles, <i>J. Gen. Physiol.</i>, 1975</p>
Image Processing (IP) & Computer Vision	<p>Light-Sensitive BZ reaction - Kuhnert et al.: <i>Nature</i>, 1986, 1989</p> <p>Autowave Principles - Krinsky et al., <i>Physica D</i>, 1991</p>	<p>Edge Detection & Segmentation - Nomura et al., <i>J. Phys. Soc. Jpn.</i>, 2003 - Kurata et al., <i>Phys. Rev. E</i>, 2009</p> <p>Stereo Disparity Detection - Nomura et al., <i>Mach. Vis. Appl.</i>, 2009</p>
Cellular Neural Network (CNN)	<p>Autowaves for IP with Chua's Circuit - Perez-Munuzuri et al., <i>IEEE CS</i>, 1993</p>	<p>Turing Pattern with Chua's Circuit - Goras & Chua, <i>IEEE CS</i>, 1995</p> <p>Segmentation with FHN + LEGION* - Wang & Terman, <i>IEEE NN</i>, 1995</p>

*LEGION: Locally Excitatory Globally Inhibitory Oscillator Network

Edge Detection Algorithms & Related Topics

Algorithms / Topics	Articles
Gaussian filter (LoG or DoG filter)	Marr and Hildreth: <i>Proc. Roy. Soc. Lond.</i> , 1980
Diffusion equation	Koenderink: <i>Biological Cybernetics</i> , 1984 Sunayama et al.: <i>Jpn. J. Appl. Phys.</i> , 2000
Anisotropic diffusion	Perona and Malik: <i>IEEE PAMI</i> , 1990 Nordstrom: <i>Image & Vis. Comp.</i> , 1990 Black et al.: <i>IEEE IP</i> , 1998
Edge detection & blur estimation	Elder and Zucker: <i>IEEE PAMI</i> , 1998
Evaluation methods & others	Heath et al.: <i>IEEE PAMI</i> , 1997 Martin et al.: <i>IEEE PAMI</i> , 2004, 2011 F-measure, Precision-Recall, contour detection
Review	Ziou and Tabbone, <i>Patt. Recog. Image Anal.</i> , 1998 Basu, <i>IEEE SMC-C</i> , 2002

Motivation

- Image processing with
 - reaction-diffusion systems
 - coupled excitable elements
- Inspired by
 - Long-range inhibition in visual systems
 - DoG filter
 - Mach bands effect
 - Stationary patterns in biological systems

The FitzHugh-Nagumo Type Excitable Element

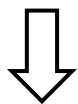
Threshold level a for binarisation

(a) Uni-stable element

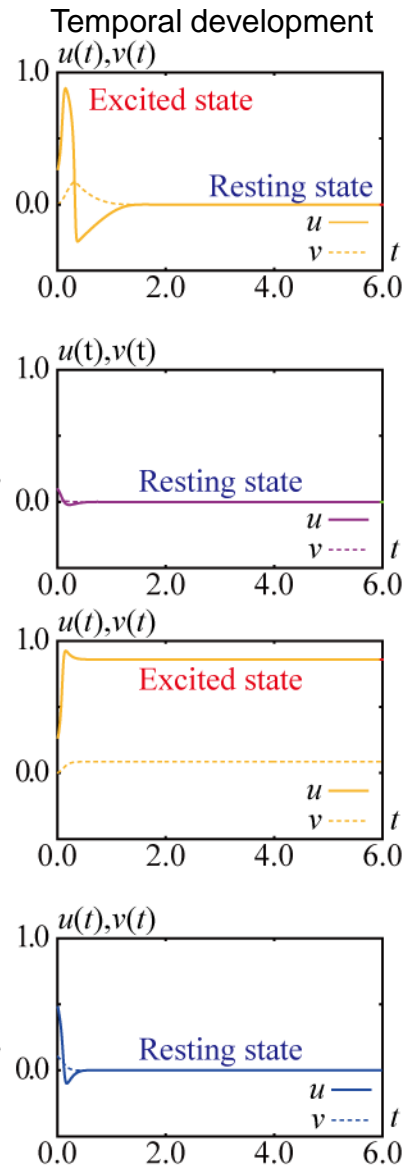
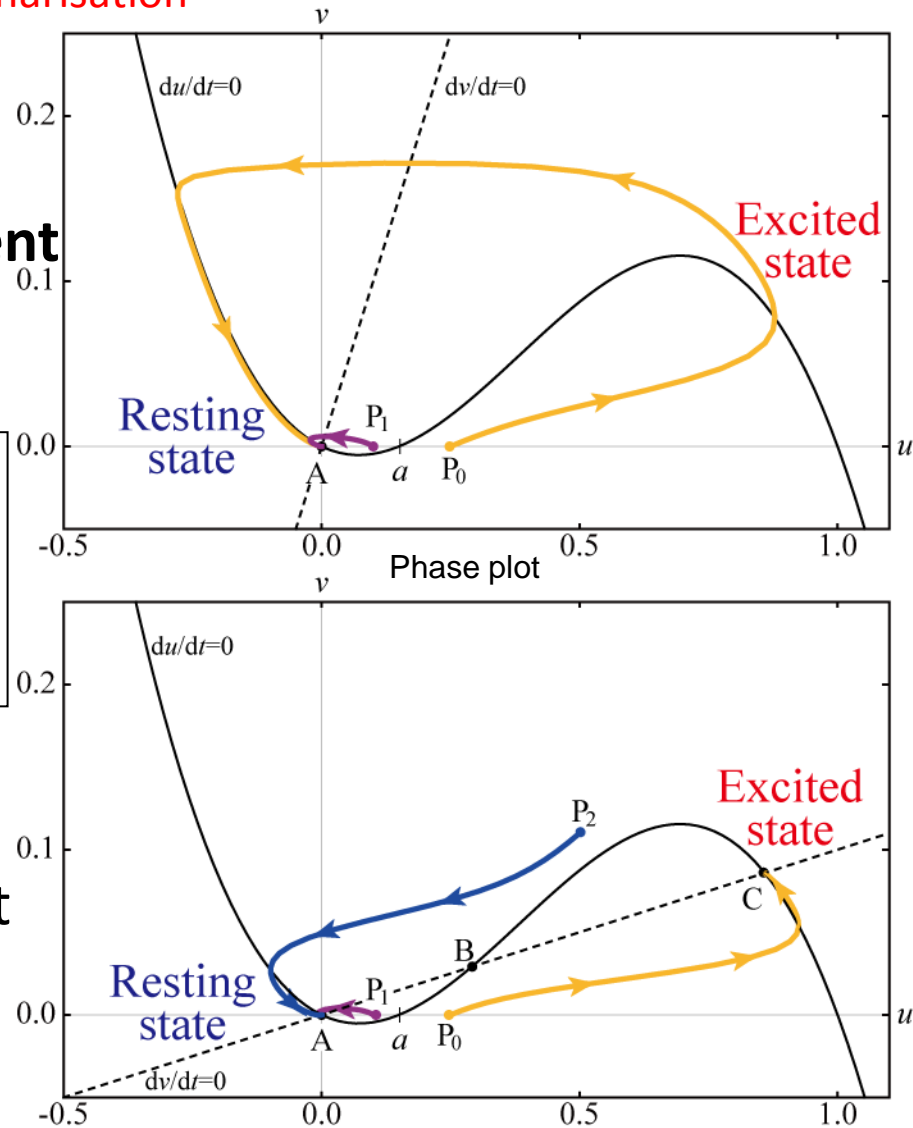


$$\frac{du}{dt} = \frac{1}{\varepsilon} [u(u-a)(1-u) - v]$$

$$\frac{dv}{dt} = u - bv$$



(b) Bi-stable element



Spatially Coupled Uni-stable Excitable Elements

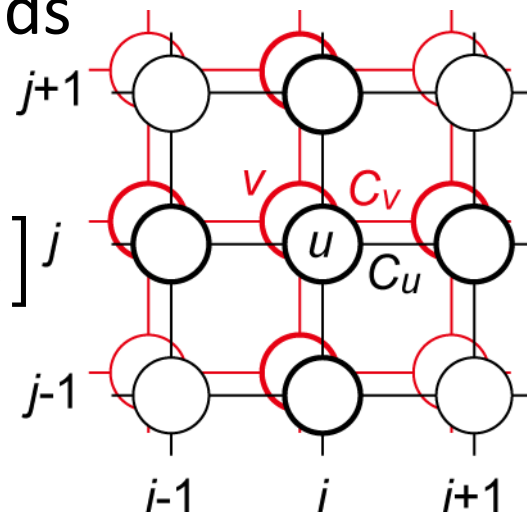
- Uni-stable elements placed at image grids

- Nomura et al., *J. Phys. Soc. Jpn.*, 2003
- Kurata et al., *Phys. Rev. E*, 2009

$$\frac{du_{i,j}}{dt} = C_u \left[\overline{u}_{i,j} - 4u_{i,j} \right] + \frac{1}{\varepsilon} \left[u_{i,j} (u_{i,j} - a)(1 - u_{i,j}) - v_{i,j} \right]$$

$$\frac{dv_{i,j}}{dt} = C_v \left[\overline{v}_{i,j} - 4v_{i,j} \right] + u_{i,j} - bv_{i,j}$$

Spatial coupling $\overline{u}_{i,j}, \overline{v}_{i,j}$: averages in the nearest four points.

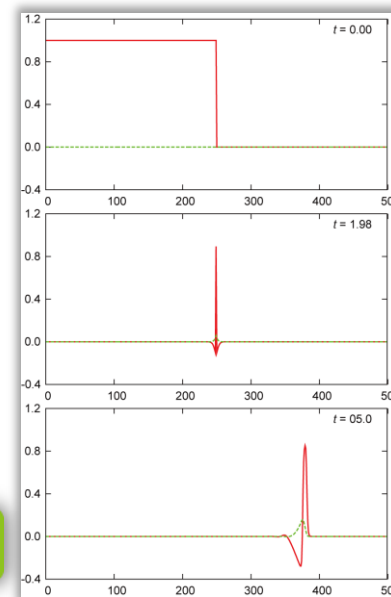


- **Strong inhibition: $C_u \ll C_v$**

⇒ Stationary pulses at edge positions

- **Weak inhibition: $C_u \ll C_v$**

⇒ Propagating pulses



Initial states
 u and v

u and v
($C_u \ll C_v$)

u and v
($C_u > C_v$)



Previous algorithm:

Examples for a Real Image

- Initial states:
 - $u_{i,j}(0)$ = “image intensity distribution” and $v_{i,j}(0) = 0.0$
- Pulses are self-organised at edges.
 - $C_u \ll C_v$: pulses are stationary at the edges.
 - $C_u > C_v$: pulses propagates and develop as spiral waves.



Real image (512×512 pixels)
Initial state of $u_{i,j}(t=0)$.



$u_{i,j}(t=10)$ with $C_v=0.0$

Movie



$u_{i,j}(t=10)$ with $C_v=20.0$

Movie

Courtesy of Heath et al.: “Edge detector comparison”,
http://marathon.csee.usf.edu/edge/edge_detection.html

Other parameter settings:
 $C_u=4.0$, $a=0.05$, $b=1.0$, $\varepsilon=1.0 \times 10^{-2}$, $\delta t=1/10000$

Edge Detection for Grey Level Image

- Two pairs of coupled elements

$$\begin{array}{l}
 \text{The 1st pair} \\
 \left\{ \begin{array}{l}
 \frac{du_{i,j}^0}{dt} = C_u \left[\overline{u_{i,j}^0} - 4u_{i,j}^0 \right] + \frac{1}{\varepsilon} \left[u_{i,j}^0 (u_{i,j}^0 - a_{i,j}^0)(1 - u_{i,j}^0) - v_{i,j}^0 \right] + \underbrace{\frac{du_{i,j}^1}{dt} \ominus \left(-\frac{du_{i,j}^1}{dt} \right)} \\
 \frac{dv_{i,j}^0}{dt} = C_v \left[\overline{v_{i,j}^0} - 4v_{i,j}^0 \right] + u_{i,j}^0 - bv_{i,j}^0
 \end{array} \right. \\
 \\
 \text{The 2nd pair} \\
 \left\{ \begin{array}{l}
 \frac{du_{i,j}^1}{dt} = C_u \left[\overline{u_{i,j}^1} - 4u_{i,j}^1 \right] + \frac{1}{\varepsilon} \left[u_{i,j}^1 (u_{i,j}^1 - a_{i,j}^1)(1 - u_{i,j}^1) - v_{i,j}^1 \right] \\
 \frac{dv_{i,j}^1}{dt} = C_v \left[\overline{v_{i,j}^1} - 4v_{i,j}^1 \right] + u_{i,j}^1 - bv_{i,j}^1
 \end{array} \right.
 \end{array}$$

- Edge map

$$M(t) = \{(i, j) \mid u_{i,j}^0 > \theta\}$$

Weak point:

Edges of narrow dark areas are not detected.

Proposed Edge Detection Algorithm

- For detecting edges of narrow dark areas,
- Utilise an original image $I_{i,j}$ and **its intensity inverted version $\neg I_{i,j}$** .
- Apply our previous edge detection algorithm to both of the images $I_{i,j}$ and $\neg I_{i,j}$.
- Obtain two edge maps $M_I(t)$ and $M_{\neg I}(t)$.
- Compute a final edge map.

$$M(t) = M_I(t) \cup M_{\neg I}(t)$$

Experimental results:

Performance Evaluation of Edge Detection Algorithms

- Tested algorithms:

- Our previous Algorithm:

- Nomura et al., *International Journal of Circuits, Systems and Signal Processing*, 2011

- Proposed algorithm

- Canny Algorithm: Canny, *IEEE PAMI*, 1986

- Anisotropic Diffusion Algorithm: Black et al., *IEEE IP*, 1998

For program codes of the Canny algorithm and the anisotropic diffusion algorithm, courtesy of Heath et al., http://marathon.csee.usf.edu/edge/edgecompare_main.html

- Accuracy measures:

- P : Precision

- R : Recall

- F -measure

$$P = \frac{1}{|M_t|} |M_t \cap M_o| \quad F = \frac{2PR}{P + R}$$

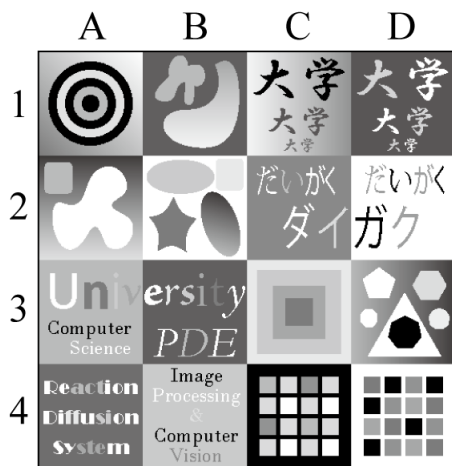
$$R = \frac{1}{|M_o|} |M_t \cap M_o|$$

M_o : obtained edge map
 M_t : true edge map

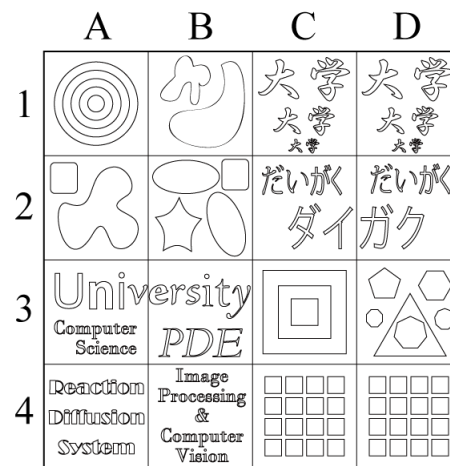
Martin et al.: *IEEE PAMI*, 2004, 2011

Experimental results:

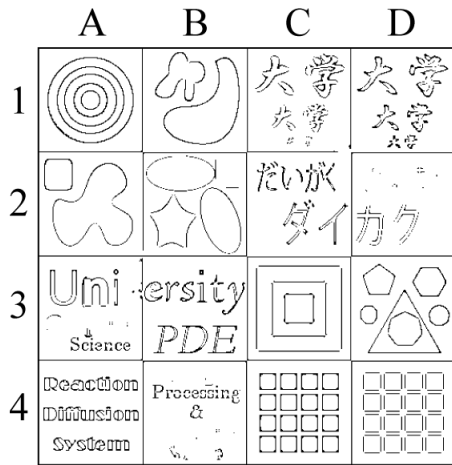
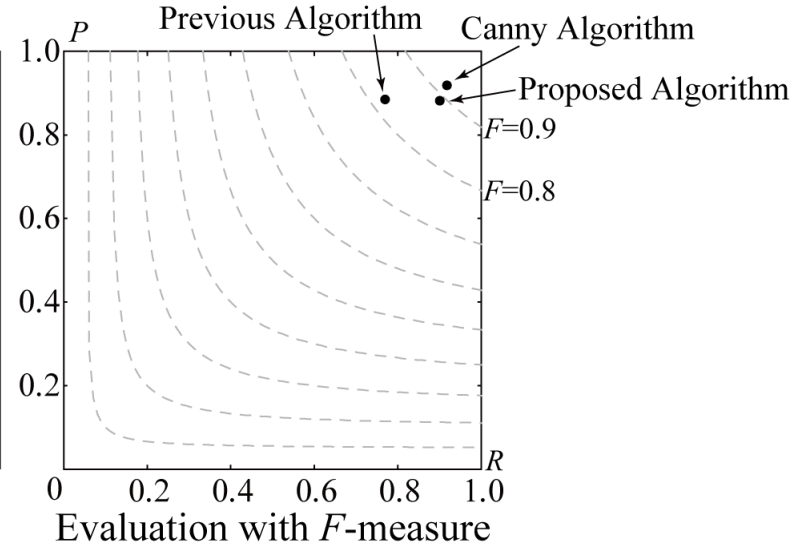
Results of Performance Evaluation for Noiseless Image



Noise-less Image



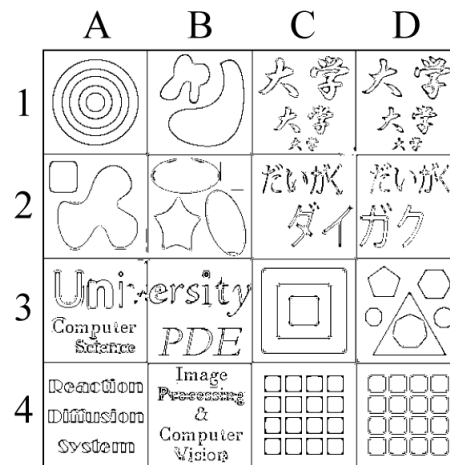
Ground-Truth



Previous Algorithm

$P=0.886, R=0.769$

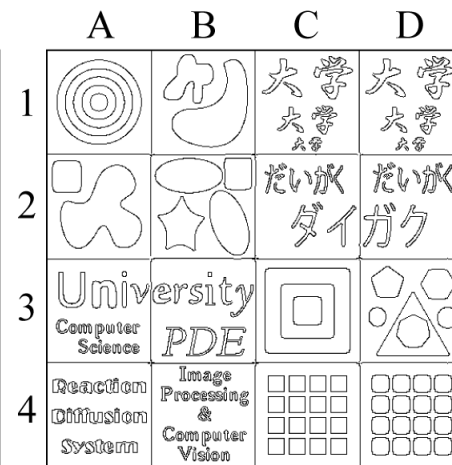
$F=0.823$



Proposed Algorithm

$P=0.883, R=0.901$

$F=0.892$



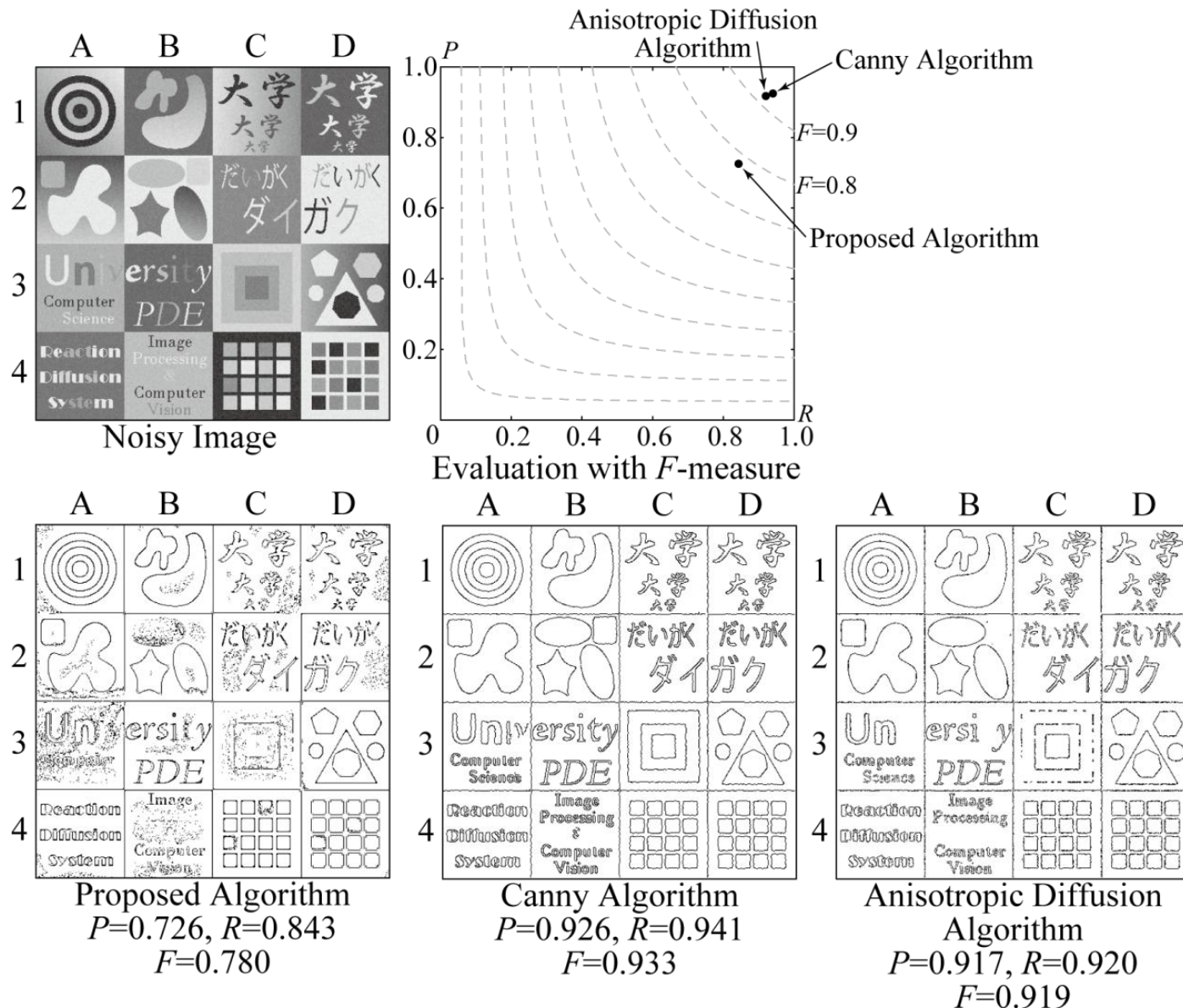
Canny Algorithm

$P=0.919, R=0.918$

$F=0.919$

Experimental results:

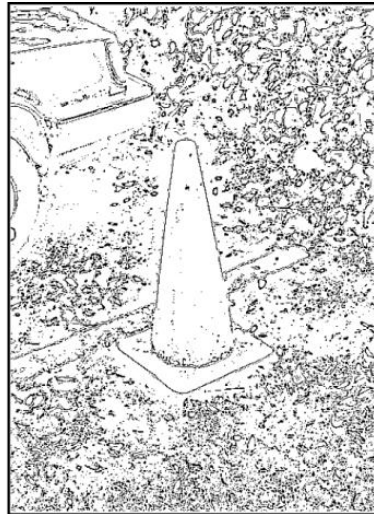
Results of Performance Evaluation for Noisy Image



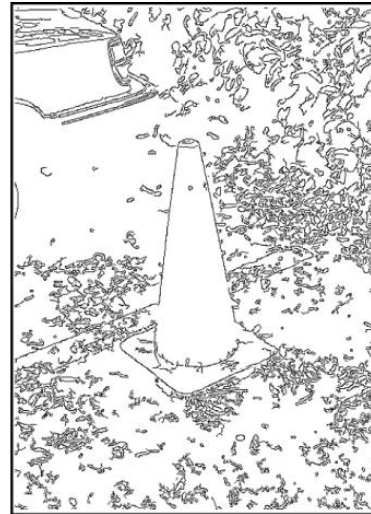
Experimental results: Results for Real Images



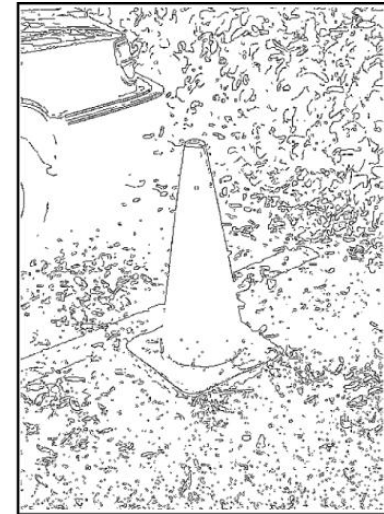
Real Image



Proposed Algorithm



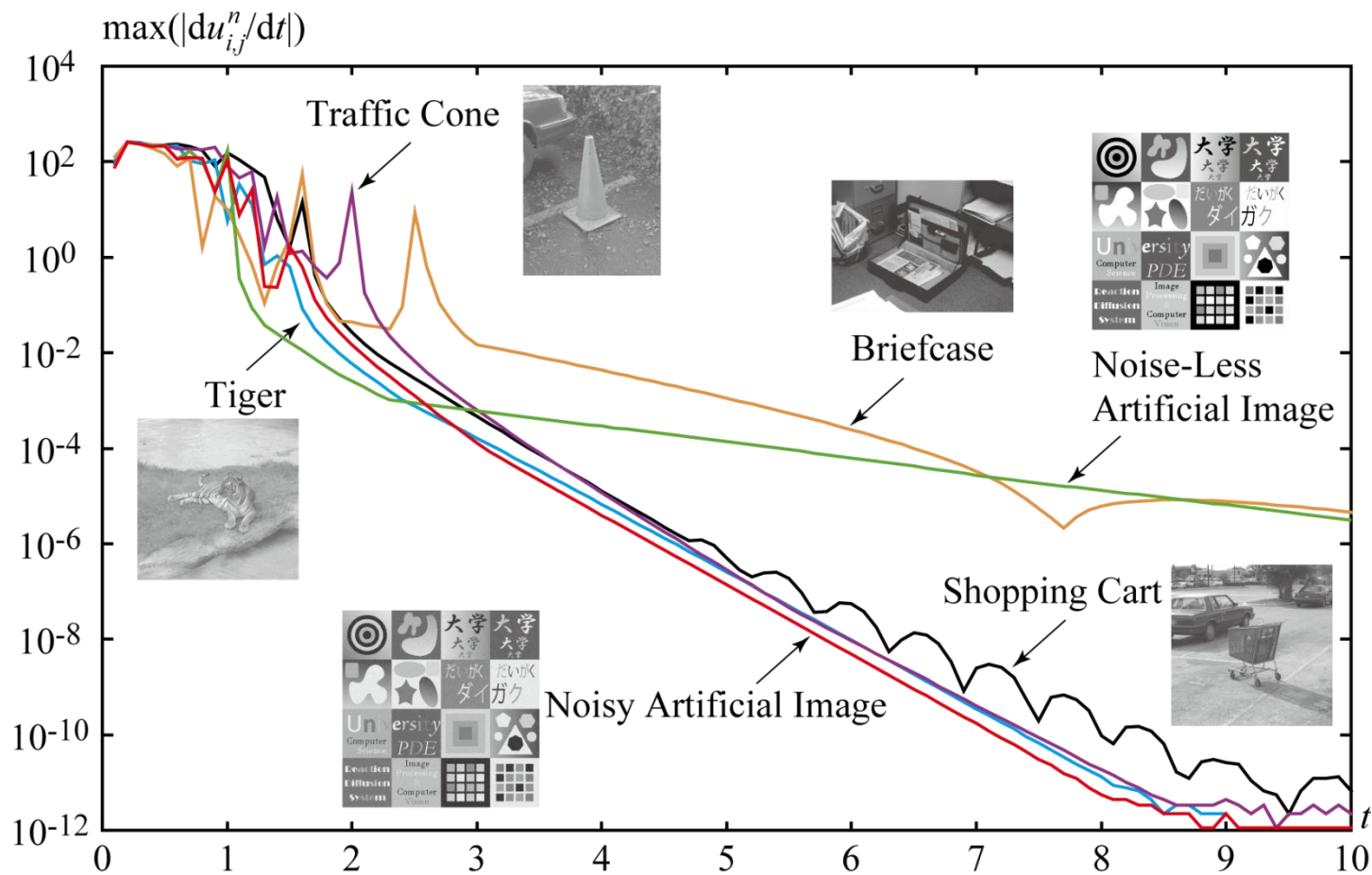
Canny Algorithm



Anisotropic Diffusion
Algorithm

[More Results](#)

Convergence of the Proposed Algorithm



Conclusion

- Edge detection algorithm for grey level image
 - FitzHugh-Nagumo excitable elements
 - Discretely spaced elements
 - Original image and its inverted version
- Experimental results
 - Artificial and real images
 - Comparison with the Canny algorithm and the anisotropic diffusion algorithm
 - Convergence

Thank you for your attention!

Any Question ?

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